[5] All producers' goods of the same vintage are exactly the same.
[6] Labour force is assumed to be constant.

The following notation is assigned in the analysis of economic life of capital goods in the firm producing consumers' goods (and used throughout the paper):

*Variables*

| | |
|---|---|
| A(O) | present net worth of an endless stream of all future acquisitions of producers' goods by the firm |
| L | number of men employed by the firm |
| n (V) | net worth of the acquisition of a new unit of producers' good of vintage v as seen at time v |
| P | price of consumers' goods |
| p | price of a new unit of producers' goods |
| S (v) | physical capital stock of producers' goods of vintage $v$ held by the firm |
| X | output produced and sold per annum |
| u | life-span of producers' goods |

*Parameters*

| | |
|---|---|
| a | labour coefficient in the producers' goods industry |
| b | physical capital coefficient in the consumers' goods industry |
| e | Euler's number, the base of natural logarithms |
| $\eta$ | price elasticity of demand for consumers' goods faced by the firm |
| $i > 0$ | interest rate per annum |
| $\mu < 0$ | proportionate rate of technological progress (in labour) per annum |
| N | multiplicative factor in the demand function |
| w | money wage |

The constant-elasticity demand function for firm's output at time $v$ is:

$$\mathbf{X}(v) = N(v)\,[P(v)]^{\eta} \tag{1}$$

where $-\infty < \eta < 0$, $N(v) > 0$, $P(v) > 0$ and $X(v) > 0$.

H. Brems took advantage of constant capital-labour ratio in consumers' goods industry (as implied by assumption [3] regarding pattern of technological progress in the model) and defined a physical unit of producers' goods as equipment operated by one man:

$$\frac{S(v)}{L(v)} = 1. \tag{2}$$

Assuming [5] the physical capital coefficient for newly built producers' goods of vintage $v$ can be defined:

$$b(v) = \frac{S(v)}{X(v)}. \tag{3}$$

And the technological progress manifests itself in a steady reduction of so defined technical coefficient:

$$b(t) = e^{\mu(t-v)}\, b(v), \tag{4}$$

where $v < t$ and $\mu < 0$. Although being the effect of technological progress, steady price reduction during the useful life-span of capital goods $v < t < v + u$ is interpreted as the firm's price policy against entry of potential competitors:

$$P(t) = e^{\mu(t-v)}\, P(v),\; v < t < v + u. \tag{5}$$

During the entire useful life $u$ of producers' goods of vintage $v$ capital coefficient remains frozen, so because of (5) the revenue per annum per unit of physical capital $P(t)/b(v)$, $v < t < v + u$ is steadily declining. As according to (2) the number of physical units of capital is always equal to the number of units of labour i. e. $S(v) = L(v)$, constant positive annual money wage per man $w$ also represents operating labour cost per annum per physical unit of producers' goods. Constancy of capital-labour ratio suggests that only technological progress *in labour*[3] (manifesting itself through rising output of consumers' goods per annum) is included in Brem's model. Positive allocative rate of interest $i$ determines exogenously the marginal capital cost to the firm, which si independent of the amount of borrowed funds. Thus in conjunction with [4] it is not difficult to arrive at the definition of the *net* worth of the acquisition of a new unit of producers' goods of vintage $v$ as seen at time $v$:

$$n(v) = \frac{P(v)}{b(v)}\,\frac{1-e^{(\mu-i)u}}{i-\mu} - w\,\frac{1-e^{-iu}}{i} - p, \tag{6}$$

where price of a new unit of producers' goods is nothing but investment cost. This cost is in line with [1] defined regardless of vintage by:

---

[2] As H. Brems didn't make this assumption explicitely, the pattern of technological progress is defined in accordance with the classification suggested by HORVAT, B. (1987), p.p. 241—260 and (1991). Neutral II technological progress (at the same rate) reduces technical coefficients exclusively in the consumers' goods industry.

[3] The emphasis is placed on this for the purpose of subsequent derivation of some basic theoretical results concerning the relation between economic and technological life of capital goods.

$$p = aw. \tag{7}$$

Its constancy during the useful life of a capital good of any vintage is self-understood. Over vintages it is constant as a consequence of the pattern of technological progress [3]. From (3) and (4) is found that the revenue per annum per unit of physical capital is also constant over vintages:

$$\frac{P(t)}{b(t)} = \frac{P(v)}{b(v)}. \tag{8}$$

Taking this into account after being modified by (7) the expression (6) implies that:

$$n(t) = n(v). \tag{6'}$$

At time t = 0 let a firm acquire the vintage zero capital stock S (O) = b(O) X (O), which is then replaced forever every *u* years maintaining constant (net output)[4] capacity. At time t = ju the physical capital stock required for the j-th replacement will be:

$$S(ju) = e^{j\mu u}\, b(O)\ X(O). \tag{9}$$

Notice that the real cost of replacement (measured in physical units according to (2)) is declining due to the increasing efficiency ot a physical unit of capital i. e. S (O) > S (ju) as $\mu < O$. If we use the equations (6) through (9) the *present* net worth of an endless series of all future acquisitions of capital goods S (O), S (u), S (2u), ... is:

$$A(O) = \frac{b(O)\, n(O)\, X(O)}{1 - e^{(\mu - i)\, u}}. \tag{10}$$

The expression (10) can be further modified by using (1) and (6) for v = O.

The firm should therefore *irrespective of market structure* adjust initial price P (O) and useful life *u* to solve the problem:

$$\underset{P(O),\, u}{MAX}\ \ A(O). \tag{11}$$

The resulting solutions for optimum (unique and positive) values, which satisfy first and second-order conditions for local maximum of the present net worth for a firm having infinite investment horizon, are given below:

[4] The additional specification will be clarified in the analysis of 'full wage costs coverage' approach.

$$P(O) = \frac{\eta}{1+\eta} \frac{i-\mu}{i} \frac{1-e^{-iu}+ai}{1-e^{(\mu-i)u}} b(O)\, w\,, \tag{12}$$

$$ie^{-\mu u} - \mu^{-iu} = (ai+1)(i-\mu). \tag{13}$$

The main result of Brems is reproduced here as equation (13). As in his model the rate of interest is determined *exogenously* (and may therefore in the analysis of the firm equilibrium vary arbitrary), the equation is transcendental one permitting no explicit solution for $u$.

Inserting (13) into (12) optimum price P (O) can be expressed as a function of optimum economic life for capital goods in Brems' model:

$$P(O) = \frac{\eta}{1+\eta} e^{-\mu u} \mathrm{b}(\mathrm{O})\, \mathrm{w}. \tag{14}$$

For the purpose of subsequent comparative analysis the consequences of Brems' approach for determination of economic life for capital goods are the best summarized by the expression (14).

## 2. FULL WAGE COSTS COVERAGE ONLY

The second approach, which was proposed by B. Horvat[5], will now be reproduced under the assumptions of Brems' model described in the previous section. The introduction of some new concepts and definitions is therefore necessary. (List of symbols additionally introduced in the equations (15) through (41) is given in the Appendix). With the constant labour force for neutral II technological progress under embodiment hypothesis we have:

$$N = e^n = 1,\ n = 0, \tag{15}$$

$$M_K = M,\ M_K = e^{-\mu_K} \text{ and } M = e^{-\mu} \ for\ \mu_K,\ \mu < O \tag{16}$$

where $n$ is the rate of growth of the *number* of (changing) producers' goods, while $\mu_K$ denotes the rate of simultaneous changes in their *efficiency cum size* and let $\mu$ be the rate at which the output of (standard) consumers' goods expands.[6] All rates are assumed to be constant.

[5] HORVAT, B. (1991). A complete list of symbols for the equations (15) through (41) is given in the Appendix.

[6] Such a result is obtainable also from Brems' vintage model of growth (BREMS, H. (1968), pp. 473—503) assuming zero growth rate of labour force (see especially equations nos. 34, 35). This model is based upon microeconomic foundations regarding the determination of economic life for capital goods, which were described in the first chapter of this article.

At $t = 0$ the net output capacity of the economy (X) is related to the number of units of capital by the proportionality constant $\kappa$ (O) as given below:

$$K(O) = \kappa(O) X(O). \tag{17}$$

As the net output expands by factor $M$, so must capital stock (K) now measured in terms of net output capacity (NOC):

$$K(t) = (NM_K)^t K(O) = \kappa(O) X(O) M^t, O \leqslant t \leqslant u. \tag{18}$$

The same number of workers operating *the same* number of producers' goods (according to condition (15) which is compatible with 2()) produce M times more baskets.[7] Keeping this in mind the factor N may be omitted in further analysis. Notice, that a plant of vintage $u$ is substituted for $M_K$ plants of zero vintage without changing the net output capacity. This is completely in line with Brems' solution to replacement problem taking the shape of expression (9). The question emerges, if Brems' choice of unit of account for (changing) capital goods is then really the most convenient one. According to (2) he decides for "unit of labour", while B. Horvat chooses "consumers' good" at $t = 0$ as defined by (17). The former changes in time as the labour productivity increases in the model. But the latter is invariant and thus theoretically acceptable. Besides, Brems' choice is inevitably accompanied by neutral II technological progress, which seriously restricts the validity of his conclusions.

Since labour remains unchanged, the output of consumers' goods per unit of labour expands by factor M. In other wors, efficiency factor M measures *global* technological progress, which manifests itself in a steady growth of the real wage ($w_r$):

$$w_r(t) = M^t \frac{X(O)}{L} = M^t w_r(O), \ t > 0. \tag{19}$$

For the value of the output of producers' goods industry remains constant (pS = const.), i. e. the shares of living and embodied labour in final output do not change[8], M is also the efficiency factor for the

[7] The physical capital coefficient defined in terms of *NOC units* would be:

$$\kappa(t) = \frac{K(t)}{X(t)} = \frac{K(O) M^t}{X(O) M^t} = \kappa(O), \ t > O. \tag{3'}$$

Its constancy is a positive proof that the technological progress occurs only in labour (see also note № 3). Note that b (v) is declining as a consequence of (2), i.e. differently defined physical unit of producers' goods.

[8] There are some strong suggestions that in actual economies the proportions of living and embodied labour remain approximately constant.

consumers' goods industry (due to [1] and (7) in Brems' model this is irrelevant for the producers' goods sector).

We will see under the heading "Obsolescence", that the approach proposed by B. Horvat is much more straightforward and enables us to arrive at the *efficient* solution for economic life of capital goods directly as the rate of interest is determined endogenously in his model; Besides, it makes possible to derive some *basic theoretical results* concerning the relation between economic and technological life of capital goods. To put these results on firm grounds on the one hand, and to highlight necessary extensions and/or modifications of Brems' model, we shall now define capital stock, new investment and replacement (in NOC units). Underline at the beginning that the number of (changing) capital goods is assumed to be the same, while their net output capacity will be different.

### 2.1. *Capital stock, New Investment and Replacement*

Assuming initial unit investment, the effective capital stock consists of all gross investments in the last *u* years K (u); notice that for $M_K = 1$ this it the number of currently operating capital goods of *equal* efficiency):[9]

$$\int_0^u M_K^v \, dv = \int_0^u e^{-\mu_K v} \, dv = \frac{1}{-\mu_K}\left(e^{-\mu_K u} - 1\right) =$$

$$= \frac{1}{-\mu_K}(M_K^u - 1) = \hat{K}(u), \mu_K < 0. \tag{20}$$

As each vintage *v* consists of one capital good (in accordance with (2) and (15)), capital stock consists of *u* capital goods and average net output capacity per capital good at t = u is:

$$\frac{\hat{K}(u)}{K} = \frac{M^u - 1}{-\mu u} = \frac{\nu}{u} \tag{21}$$

For t > u obviously *K* remains constant, but K (t) (as defined by (18) in conjunction with (15) and (16)) expands by factor *M*, what implies that output per capital good increases by the same factor.

New investment at t = u, marked as $n\hat{I}$ (u), is by definition the difference between gross investment and replacement. The latter is the scrapped capital good or alternatively, gross investment at t = 0.[10]

[9] All categories measured in NOC units will be henceforth denoted by the cap above the belonging sign.

[10] Evidently, here it will do to analyse the first replacement only.

$$M_K^u - 1 = n\hat{I}(u) \tag{22}$$

Since effective capital stock and net output of the system expand at the same rate, i. e. $\hat{\mu}_k = \mu_x$, the *global* rate of growth given by the ratio of new investment (22) and capital stock (20 is:[11]

$$\mu^u = \frac{M_K^u - 1}{\dfrac{M_K^u - 1}{-\mu_K}} = -\mu_K, \mu_K < 0. \tag{23}$$

Output of consumers' goods, capital goods and productivity may generally change at different rates. Therefore the rate relevant for price formation and investment evaluation shall be determined.

As the money wage rate is the numéraire of the price system:[12]

$$w = P(t)\, w_r(t) = 1, \tag{24}$$

in a steady-state full-employment equilibrium H. Brems arrived (approximately) at *labour* prices. Labour prices are cost prices, which consist of labour costs and capital costs. Labour costs are determined by technology exclusively $\{\lambda(t), \lambda_{\hat{s}}(t)\}$, where the labour coefficient in the consumers' goods industry at $t = 0$ is:[13]

---

[11] In order to stay strictly within Brems' model, by definition negative rate $\mu_k$ is retained, albeit this is inconvenient. It seems to me, that Brems found this suitable only because of the declining b (t) and P (t) in expressions (4) and (5), respectively.

[12] BREMS, H. (1968), p. 475. Notice, that constancy of w is the result of P (t) and $w_r$ (t) moving in the opposite directions as indicated by (5) and (19), respectively.

[13] In original Brems' model labour coefficient could be — taking into account (2) and (3) — defined only with respect to capital good of certain vintage v:

$$\lambda(v) = \frac{L(v)}{X(v)} \tag{25'}$$

and it would coincide with the physical capital coefficient (3) due to the definition of a physical unit of producers' goods. Our definition of time pattern for the real wage (19) is thus directly comparable to Brems' definition given by equation (58), BREMS, H. (1968), p. 497.

For the purpose of comparison of the two approaches, it is useful to make another remark. Defining capital in NOC units (25) and (3) no longer coincide. Besides, the labour coefficient in the producers' goods industry is by analogy with (25):

$$\lambda_{\hat{s}}(O) = \frac{1}{\hat{S}}, \tag{25''}$$

$$\lambda(O) = \frac{L}{X(O)}. \tag{25}$$

As a consequence of the technological progress it is steadily declining over the useful life of capital goods:

$$\lambda(t) = \lambda(O)\, M^{-t},\ 0 \leqslant t \leqslant u. \tag{26}$$

Capital costs consist of replacement and new investment necessary to maintain full employment of labour force. As the labour force (according to [6]) remains constant, capital costs also depend only on technology $\{\kappa(t), u\}$. In the stationary economy, where the rate of technological progress is reduced to $\mu = 0$, the replacement costs are $\frac{1}{u}K$.

In our model with constant labour force and changing technology *replacement* is still $R(t) = I(t - u)$, where $I$ stands for gross investment and $t \in [O, u]$. Consequently, *its* current rate will depend on the rate of growth of capital (in NOC units) and also on technological progress. What has to be maintained by replacement is not physical machines but net output capacity[14], which expands by factor $M_K$ as follows from (23). Current replacement for initial unit investment now starting at some arbitrary point of time amounts to:

$$\hat{R}(t) = \hat{I}(t - u) = M_K^{(t-u)},\ t \in [O, u] \tag{27}$$

and current output capacity is equal to:[15]

$$\hat{K}(t) = \int_{t-u}^{t} M_K^{v}\, dv = \frac{1}{-\mu_K} M_K^{(t-u)} (M_K^{u} - 1). \tag{28}$$

---

where $\underline{1}$ denotes constant employment in this industry. As the number of capital goods currently generated is $\hat{S} = M_k^u$ in NOC units (while it is $S = N^u - 1$ in physical units), (25") also declines over $\underline{u}$:

$$\lambda_6^{\wedge}(t) = \lambda_s^{\wedge}(O) M^{-t},\ O \leqslant t \leqslant u. \tag{26"}$$

Having in mind that $\kappa(t) = \kappa(O)$ (see note № 7), we can sum up: Evidently, time patterns of technical coefficients in NOC units, (i.e. $\lambda$, $\kappa$, $\lambda^{\wedge}$) reproduce Neutral $\underline{I}$ technological progress, which is considered to be a very good approximation to reality (HORVAT, B. (1987), p. 242). Therefore the *theoretical conclusions of the 'full wage costs coverage' approach are generally valid.* Such a conclusion (only seemingly) contradicts [3]. This is explained by the choice of the unit of account for (changing) capital goods. Obviously, our initial concern about Brems' choice (see p. 7, the first paragraph) was well grounded.

14 See note № 4.

15 For $t = u$ expression (28) reduces to (20). Notice also, that under the embodiment hypothesis the net output rate of growth must be used when the replacement rate is calculated. (HORVAT, B. (1991), p. 118) Here, in accordance with (16) $\mu_k = \mu$.

Replacemnt per unit of capital is then:

$$\frac{1}{\nu} = \frac{\hat{R}(t)}{\hat{K}(t)} = \frac{-\mu_K}{M_K^u - 1}. \tag{29}$$

As the number of capital goods remains constant (15), the ratio $\frac{1}{\nu}$ is only a function of changes in their efficiency and size. For constant labour force the following relation holds in general: $\nu \geqslant u$ and equality applies only when there is no technological progress (M = 1).

Note, that under the embodiment hypothesis [3] *u* need not be equal to physical life-span of capital goods but may be truncated. If being so, more capital goods will have to be replaced by new, more efficient ones and that is why total output will rise. But *truncation* is worthwhile only if *net* output also increases. According to (29) the shorter is *u*, the greater is replacement ratio (what increases capital costs).

We can now define the rate of gross investment (gross profit rate), which is in our model determined in the consumers' goods industry. According to (29) and (23) it can be written as:

$$r = \frac{1}{\nu} - \mu_K,\ \mu_K < 0. \tag{30}$$

For the latest vintage (v = t) the missing price equation at the level of a *representative* firm (i. e. firm with average distribution of capital goods over all *u* vintages in existence) producing consumers' goods is then:

$$rp\kappa + w\lambda = P, \tag{31}$$

which is the only one interesting for our purpose.

Note, that all labour (capital) coefficients formerly used must be 'full' or vertically integrated. So the difference between 'full' and direct coefficient is labour (capital) content of material costs. Claim for *full coverage* of growing wage costs therefore includes also those contained in reproduction materials. While full capital costs are given during the useful life by definition and this holds irrespective of the sort of output produced.[16]

[16] As the two industries are fully integrated [1], the problem of 'double counting' with respect to material costs is thus avoided.

For the purpose of comparison of the two approaches to the determination of economic life for capital goods is suggestive to put down the *net* profit rate:

$$\pi = -\mu_K,\ for\ \mu_K < 0. \tag{32}$$

Remember that in Brems' model interest rate is explained exogenously. As the net profit rate here plays the role of the allocative rate of interest and (16) holds, the latter should be explained endogenously (i. e. within the model) by the rate of growth, what relates to the Phelps' Golden Rule of Accumulation:[17]

$$i = -\mu,\ \mu < 0. \tag{33}$$

On this basis it can be shown, that in Brems' model maintaining constant net output capacity forever the higher the rate of growth, the shorter is the optimum useful life due to the *replacement effect*. Interpret the interest rate as the rate of growth[18] and define transformative factor β by the ratio of real capital costs in the uniformly growing system to those in the stationary system using (9) for the first replacement only (j = 1):

$$\beta = \frac{S(u)}{S(O)} = \frac{e^{-iu}\, b(O)\, X(O)}{b(O)\, X(O)} = e^{-iu}. \tag{34}$$

Transformative factor β has the following characteristics:[19]

$$\lim_{i \to 0} \beta = 1, \qquad \lim_{i \to \infty} \beta = 0, \qquad \frac{d\beta}{di} < 0. \tag{35}$$

Evidently, the higher is the rate of growth, the lower is the real capital cost (what is suggested also by (29)). This implies high wage costs for given price. β depends only on *iu* and by analogy the relations (36) must hold.

$$\lim_{u \to 0} \beta = 1, \qquad \lim_{u \to \infty} \beta = 0, \qquad \frac{d\beta}{du} < 0. \tag{36}$$

Notice that u = 0 corresponds to the reproduction materials. And that the real capital costs converge to zero when useful life is approaching infinity.

Finally, after the analysis of Brems' model from the point of view of the labour theory of prices, which indicates the compatibility of the

[17] PHELPS, E. (1961).
[18] HORVAT, B. (1987), pp. 145—146.
[19] HORVAT, B. (1973), pp. 355—356.

two approaches, we can briefly indeed state the essence of the approach suggested by B. Horvat.

### 2.2 *Obsolescence*

To avoid losses, technological life of older, less efficient, capital goods should be truncated. Irrespective of the pattern of technological progress, production will be profitable only if the price — which is uniform for all vintages existing at time (t — u) — covers full wage costs:

$$P(t) \geqslant w\lambda(t-u), \tag{37}$$

while capital costs are overhead. The inequality (37) is a direct counterpart to (13) in the original Brems' model, as it tells us when an old unit of producers' goods has to be *replaced.* While under the macroeconomic approach the only relevant question is: *"When will the production become unprofitable?"* As labour coefficient is declining over the useful life of capital good [t — u, t] by factor $M_\lambda$ (where subscript $\lambda$ stands for technological progress *in labour)* and $\lambda(t)$ belongs to the latest vintage, the *efficient* solution for economic life of capital goods ($u^*$) is according to B. Horvat determined by the inequality:

$$P(t) < w\lambda(t)M_\lambda^{u^*}. \tag{38}$$

In other words, as the real wage rate (19) grows steadily from the point of view of the global efficiency a capital good must be replaced *when (constant) output price ceases to cover full wage costs.* By assumption every firm uniformly increases labour productivity and that is why a new unit of producers' goods is *always* installed exactly when u* years goes by. Notice that under the Brems' approach it pays to install a new unit of producers' goods earlier if a firm has faster technological progress (than the representative firm has). Such a possibility is under the macroeconomic approach ruled out by definition. The 'decision rule' (38) is therefore the main result of the Horvat approach.

Notice also, that the interest rate in the condition (38) is precisely the rate of uniform, embodied technological progress. Namely, as the macroeconomic equilibrium implies the labour theory of prices (31), it follows that $\mu_\lambda = \mu$ ($\mu_\lambda < 0$). Taking into account (33) we can write that $\mu_\lambda = -i$. And by analogy to the definitions in the condition (16), the growth factor of the full wage costs is defined:

$$M_\lambda = e^{-\mu_\lambda} = e^i \text{ for } \mu_\lambda < 0. \tag{39}$$

Inserting (39) into (38) we get:

$$P(t) < w\lambda(t)\,e^{iu^*}, \tag{38'}$$

from which the explicit solution for u* is (contrary to (13)) obtained in the presence of the interest rate — *now having the equilibrium value* — by simple algebraic manipulations. As in the Brems' model $i = -\mu$ only by chance, the solutions to (38') and (13) are expected to differ in general (i. e. $u^* \neq u$). Having in mind that the two approaches are compatible the conditions, under which the economic life for capital goods is unique (i. e. $u^* = u$), are to be specified.

Before we go on, notice that the criterion (38) suggests the following generalizations.[20] Namely, the economic life will be *reduced* in labour si fast (i. e. $M_\lambda$ is large). Secondly, wage costs are high (high relation to the technological life if: Firstly, technological progress in $w\lambda(t)$ implies low capital costs as P is given). Thirdly, technological durability of producers' goods is long (high *u* implies low replacement ratio $\frac{1}{\nu}$ as defined in (29)). Fourthly, rate of growth is high. Fifthly, truncation (reduction of *u*) increases replacement costs and price of consumers' goods (P is specified in (31)). And sixthly, fast technological progress in capital (i. e. high $M_K$) reduces production of capital goods relative to production of consumers' goods and consequently prices fall.

## 3. COMPARISON OF THE TWO APPROACHES

According to the pure labour theory of prices and interest "in a model with constant labour force and technological progress (assumptions [6] and [3] respective) the interest rate is precisely the rate of uniform technological progress".[21] Therefore the relation (33) is a sound modification of Brems' model. Besides, in the special case of pure competition the price elasticity of demand for consumers' goods faced by the *representative* firm will approach minus infinity ($\eta \rightarrow -\infty$), and the following approximation can be used:

$$\frac{\eta}{\eta + 1} = 1 \qquad (40)$$

in the expression (14) to obtain *efficient* solution for *u:*

$$P(O) = e^{-\mu u} b(O) w. \qquad (14')$$

Notice, that when compared with (14) in original Brems' model the optimum is generally changed.

[20] HORVAT, B. (1991), pp. 119—120.
[21] HORVAT, B. (1987), p. 267.

Furthermore, the labour coefficient coincides with the capital coefficient[22], i. e. $\lambda (v) = b (v)$. Besides, technological progress occurs only in labour and taking into account (39) $M_\lambda = e^{-\mu\lambda}$ · Therefore (14') can be written as:

$$P (O) = w\lambda (O) M_\lambda^{\ *} \tag{41}$$

Evidently, the condition (41) closely corresponds to the criterion (38) for $t = 0$, which was set forth under alternative approach proposed by B. Horvat. Notice also, that Brems' model allows for the first (large $M_\lambda$ ), second (high $w\lambda (O)$) and fourth (high growth rate) generalization regarding truncated physical life-span of capital goods. Notice, that taking into account (38') the effect of the interest rate upon the economic life is on the one hand subsumed in the first generalization,[23] while on the other hand this effect should not be treated separately as *in the macroeconomic equilibrium* the interest rate cannot vary arbitrary.

In the 'present net worth maximization' approach the attention is focused on the *microeconomic profitability* and so we do not necessarily arrive at the efficient solution for the useful life. This is characteristic of the micro- macro link, which is inherent in Brems' approach. While 'full wage costs coverage' approach is concentrated on the *global efficiency* of the economy. Because of so established macro-micro link B. Horvat always obtains the efficient solution directly. The 'full wage costs coverage' approach is evidently much more straightforward and can be considered a theoretically preferred approach. Having this in mind, the main results of our analysis can now be summarized in the following theoretical hypothesis:

Under the pure competition with the allocative rate of interest being determined by the rate of neutral, embodied technological progress, the economic life of capital goods will be the same *whether* a firm with infinite investment horizon maximizes present net worth *or* replaces a capital good exactly when price ceases to cover (growing) full wage costs.

In the real world market competition is far from being pure. Therefore in the absence of macroeconomic coordination of investment decisions a private firm is generally not in a position to arrive at optimal economic life for capital goods. If the net output growth ought to be maximized[24], we may draw the following, *pure theoretical conclusion:*

---

[22] See note № 13 for explanation.

[23] We have found out that $\mu_\lambda = \mu$ and according to (33) $-\mu = i$. As by definition the rate of global technological progress is negative ($\mu < O$) while the rate of interest is positive ($i > O$), they have *the same* effect upon optimal useful life. Remember that in accordance with (16) $\mu = \mu_k$ and see also note № 11 for explanation.

[24] i.e. the output of consumers' goods per unit of labour ought to expand by factor M. HORVAT, B. (1965), str. 575.

Economic planning to determine the allocative rate of interest proves to be a necessary precondition for optimal solutions.

In other words, if the global efficiency of the economy is to be kept through the investment process, it will have to be actively pursued as an explicit aim of economic policy.[25] The problem of practical implementation is naturally outside the domain of the pure theoretical analysis.

*Received: 16. 04. 1991*
*Revised: 05. 09. 1991*

## QUOTED LITERATURE

BREMS, H. (1968), Quantitative Economic Theory: A Synthetic Approach, John Wiley and Sons, New York.

HORVAT, B. (1965), The Optimum Rate of Investment Reconsidered, Economic Journal, Vol. 75, Sept., str. 572—576.

HORVAT, B. (1973), O optimalnoj alokaciji resursa u vremenu, Ekonomist, Vol. 26, № 3—4, pp. 351—360.

HORVAT, B. (1978), Radna teorija cijena, Rad, Beograd.

HORVAT, B. (1991), Embodied and Unembodied Technological Progress Compared, University of Zagreb, Mimeo.

PASINETTI, L.L. (1981), Structural Change and Economic Growth, McMillan Press, London.

PHELPS, E. (1961), The Golden Rule of Accumulaton: A Fable for a Growthmen, American Economic Review, pp. 638—643.

APPENDIX: List of symbols *additionally* introduced in the equations (15) through (41)

| | |
|---|---|
| $\hat{I}(t)$ | gross investment at time *t* in NOC units |
| $I(t)$ | gross investment at time *t* |
| $K$ | capital stock (number of producers' goods) |
| $K(t)$ | capital stock at time *t* measured in terms of net output capacity |
| $\hat{K}(u)$ | effective capital stock |
| $l$ | number of workers employed in the producers' goods industry |
| $M = e^{-\mu}$ | factor of the net output expansion |
| $M_k = e^{-\mu_K}$ | factor of the efficiency cum size changes in producers' goods |
| $M_\lambda = e^{-\mu_\lambda}$ | factor of labour productivity growth |
| $N = e^n$ | multiplicative factor for the number of (changing) producers' goods |

[25] PASINETTI, L.L. (1981), pp. 90—91; drew the same conclusion with regard to the full employment.

| | |
|---|---|
| $n$ | the rate of growth of the number of changing producers' goods |
| $n\hat{I}(u)$ | new investment at time t = u in NOC units |
| $R(t)$ | replacement at time *t* in physical units |
| $\hat{R}(t)$ | replacement at time *t* in NOC units |
| $r$ | rate of gross investment (gross profit rate) |
| $\hat{S}$ | number of capital goods currently generated in NOC units |
| $u^*$ | efficient solution for economic life of capital goods |
| $w_r(t)$ | real wage at time *t* |
| $X$ | net output capacity of the economy |
| $\beta$ | transformative factor |
| $\kappa(O)$ | proportionality constant, physical full capital coefficient at time t = O in NOC units |
| $\kappa(t)$ | physical full capital coefficient at time *t* in NOC units |
| $\lambda(v)$ | labour coefficient in the consumers' goods industry defined with respect to capital good of vintage *v* |
| $\lambda(t)$ | physical full labour coefficient at time *t* in the consumers' goods industry |
| $\lambda_{\hat{S}}(t)$ | physical full labour coefficient at time *t* in the producers' goods industry |
| $\mu < O$ | rate of growth of the net output (i.e. production of standard consumers' goods) |
| $\mu_k < O$ | rate of simultaneous changes in the efficiency cum size of producers' goods |
| $\mu_\pi$ | global rate of growth |
| $\mu_\lambda < O$ | rate of technological progress in labour (i.e. rate of growth of labour productivity) |
| $\nu$ | replacement per unit of capital in NOC units |
| $\pi$ | net profit rate |

## EKONOMSKA KRETANJA KAPITALNIH DOBARA

*Barbara BOJNEC, rođ. FAKIN*

### *Rezime*

*U radu je pomoću modificiranog Bremsovog dvo-sektorskog modela dokazana teorijska hipoteza: „Da ako u uslovima čiste konkurencije alokacijsku kamatnu stopu određuje stopa neutralnog, opredmećenog tehnološkog progresa, onda će životni vijek kapitalnih dobara biti jednak,* ili *poduzeće uz beskonačan investicijski horizont maksimira neto sadašnju vrijednost* ili *ako uvijek zamjeni kapitalno dobro upravo kad cijena ne može više pokriti (rastuće) ukupne nadnične troškove'.*

*Pristupu ‚potpunog pokrića ukupnih nadničnih troškova' treba s teorijskog stanovišta dati prednost, jer njime uvijek direktno i na dosta jednostavniji način dolazimo do efikasnog rješenja za životni vijek kapitalnih dobara.*

*Iz uslova (41) proizilaze tri osnovna teorijska rezultata. Naime, ekonomski životni vijek se reducira* ispod *tehnološkog vijeka upotrebe kapitalnih dobara ako je ispunjen jedan od slijedećih uvjeta: Prvo, tehnološki progres rada treba da bude brz. Drugo, troškovi rada su veliki. Ili treće, stopa rasta je visoka.*

*U stvarnosti tržišna konkurencija međutim nije čista. Stoga privatno poduzeće bez sistematične opće-privredne koordinacije investicijskih odluka općenito neće odabrati optimalan ekonomski životni vijek kapitalnih dobara. Budući da bi trebalo maksimizirati stopu rasta neto proizvoda (potrošnje) proizilazi slijedeći* teorijski *zaključak. Naime, da je ekonomskim planiranjem određena alokacijska kamatna stopa nužan preduslov za optimalna rješenja. Drugim riječima, globalna efikasnost privrede u procesu investiranja će se ostvarivati jedino, ako bude aktivno praćena kao eksplicitni cilj ekonomske politike. Kakva će međutim biti praktična primjena ustanovljenog nije u domeni čiste teorijske analize.*