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NONLINEAR GOAL PROGRAMMING MODEL APPLICATION IN OPTIMAL ECONOMY DEVELOPMENT POLICY CHOOSING[1]

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ABSTRACT

In this paper a new nonlinear goal programming model (NGP) and a new interactive goal programming methodology (MINGP) have been developed which are more applicable then existent. A gradient nonlinear programming algorithm, based upon the feasible directions method, built in an optimal step length routine, has been used to develop an algorithm for NGP model solving, especially those in which nonlinear functions are of Cobb—Douglas type. MINGP is an effective means to consider the models which involve multiple often conflicting goals. Tests of this methodology in real optimal economy development decision situations were affirmative.

KEYWORDS: Econometric models, feasible directions, gradient method, nonlinear goal programming, interactive method, economy development policy, economic policy.

1. INTRODUCTION

Recently many attempts have been made to extend the field of mathematical modelling and optimization application to the field of scientific decision making. An importat contribution to the theoretical fundamentals and practical elaboration of decision making process has been made by the formalization and application of quantitative proceedings for discovering the optimal solutions to the problems of multiple criteria decision making (Johnson 1968, Lee 1972, Simon and others 1976, Johansen 1978).

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In this paper a new nonlinear goal programming model and a new interactive goal programming methodology — MINGP — have been developed which are more applicable than existing ones. Interactive MINGP methodology serves as a supporting means for decision preparation when the decision maker, which as is often the case, does not feel sure about defining the priority of weight coefficients in the objective function of a goal programming problem.

If required by the decision maker, this methodology also checks compromise solutions and if need be, structures a new objective function which may enable him to reach satisfying and reasonable solutions. MINGP has been tested on a small empirical econometric model of the Yugoslav economy. Tests of this methodology in actual decision making situations gave encouraging results.

The efficiency of MINGP is conditional on the application off up-to-date information equipment (PC, time monitor systems etc.) as only then does it represent a powerful means for decision making support. This paper gives the appropriate algorithms and computational solutions.

2. GOAL PROGRAMMING AND GRADIENT METHOD OF FEASIBLE DIRECTIONS

The goal programming method has been considered the oldest method of multiple objective programming. It has been very popular in the applicative sense but waited far too long to pass from a theoretical basis (Charnes and Cooper, 1961) into the sphere of practical application (Lee, 1972 and Ignizio, 1976).

The problem of nonlinear goal programming can be expressed in the following way:

$$Min\ F(d) = \sum_{l=1}^{k} \sum_{i=1}^{n} w_i P_l (d^+_i + d^-_i) \tag{1}$$

s.t.

$$G_i(x) = \sum_{j=1}^{n} g_{ij} x_j + h_{ij} \prod_{j=1}^{n} x_j^{e_{ij}} + d^-_i - d^+_i = c_i \tag{2}$$

$$A_i(x) = \sum_{j=1}^{n} a_{ij} x_j \leq b_i \tag{3}$$

$$x_j, d^-_i, d^+_i \geq 0, d^-_i \cdot d^+_i = 0, \forall\ i = 1, \ldots, m, j = 1, \ldots, n. \tag{4}$$

where x_j are decision making variables, d^-_i and d^+_i represent negative and positive deviational variables from the goals (under achievement and overachievement), respectively. g_{ij} are coefficients of linear portions of goals (constraints (2)), a_{ij} are coefficients of structural constraints, h_{ij} are coefficients of nonlinear portions of goals, and e_{ij} are exponents. c_i and b_i are constants of right hand sides.

P_l in objective function (1) are preemptive priority factors, so that the following is valid:

$$P_j >>> P_{j+1} \ \forall \ j = 1, 2, \ldots, k.$$

The highest priority is indicated by P_1, the next highest by P_2, and so forth. w_l are weights assigned to some priority factors. The notion of priorities holds that P_1 is preferred to P_2 regardless of any weights w_i associated with P_2.

In great majority of up-to-date developed methods used for solving problems of this very type among the most significant ones, is the gradient method combined with the feasible directions method.

The feasible directions methods have been primarily designed for nonlinear programming problems and they are the iterative ones whose solutions in certain iterations have the following recursive form:

$$x_{p+1} = x_p + l_p s_p(x) \ \forall \ p = 1, 2 \ldots$$

where $s_p(x) = s(x_0, x_1, x_2, \ldots x_p)$ is direction and $l_p \geq 0$ is a step size, which is chosen so that:

$$F(x_p + l_p s_p) \leq F(x_p),$$
$$\forall \ (x_p + l_p s_p) \in X, \ 0 \leq l_p \leq 1, \ s_p(x) \geq 0,$$

and where: $X = \{x \in R^n \mid$ conditions (2), (3) and (4) are satisfied$\}$. If l_p is chosen on this way, it follows that all points between x_p and x_{p+1} are feasible.

Gradient methods use the direction of a gradient as a solution improvement direction, thus defining the feasible and usable feasible direction and enabling reduction of the nonlinear problem to a approximate linear problem which is close to the initial one or some other by the iterative proceedings set solution.

The method is iterative and each iteration starts with an initial feasible vector. At each iteration of the feasible directions method a feasible direction of improvement (usable feasible direction) is determined and a new "better" point is found along that direction. Optimality is achieved when no further improvement can be made in any feasible direction.

To identify the gradient this method requires that functions are continuous and differentiable. In order to guarantee convergence of the algorithm the gradient method requires that the model constraints form a convex set at each goal level while the objective function is concave. Most solution methods for nonlinear programming problems are restricted to convex programming problems, that is, problems which involve minimization of a convex function or maximization of a concave function over a convex region. In our case the objective function, which is to be minimized, is linear and accordingly both convex and concave, so that it fulfils the requirement in any case. Another requirement for ensuring "good" convergence, that nonlinear constraints satisfy the so-called Slater's conditions, is also fulfilled (on conditions of feasible direction method convergence see: Zlobec and Petrić, 1989, p. 263).

The step size movement in feasible direction, the method of its computation and "density" of solutions search constitute the basic difference between some of the methods from the group of feasible directions gradient methods.

We are in this case particularly interested in a nonlinear function of the Cobb—Douglas type, the general form of which is:

$$g(x_1, x_2, \ldots, x_n) = h \prod_{i=1}^{r} x_i^{bi} \tag{5}$$

where: $x_i \geq 0$, $\forall$ $i = 1, \ldots n$, are independent variables, $h \in R$ is coefficient and $b_l \in R$ are exponents of independent variables. This is the most familiar and most frequently applied form of production function by which the production value in a certain economy is expressed as a function of labor and capital investment.

The idea of linearization of nonlinear constraints and solution of nonlinear programming problems as linearized programming problems is not new, because Griffith and Stewart first suggested in 1961 that nonlinear problem may be linearized in the region about the particular point by expansion as a Taylor's series, ignoring terms of a higher order than linear and adding two more restrictions for each nonlinear constraints. In that way nonlinear programming problems have been transformed into a form which can be solved by the linear programming algorithm (see: Griffith and Stewart 1961, Ignizio 1976, pp. 156—162, and especially Lee, S. M. and Olson D. L. 1985).

Our idea was to apply Euler's theorem for the "total" linearization of nonlinear constraints in the region about the particular point (see: Roljić, 1987). Owing to Euler's theorem of function homogeneity it is possible to apply this method to solve the problems of NGP regardless of whether Cobb—Douglas functions are linearly homogeneous, t.i $\sum_{i=1}^{n} b_i = 1$ or positively homogeneous, that is $\sum_{i=1}^{n} b_l > 0$. In economic theory, production functions are frequently assumed to be linearly homogeneous, because sush functions have convenient properties.

The advantage of our approach is that it is not necessary to add two more constraints for each linearized nonlinear constraint in every simplex iteration of algorithm, and because the linearization is more accurate.

3. ALGORITHM OF NGP AND MINGP

As already mentioned, an algorithm of nonlinear goal programming is based on hybrid connection of modified simplex method of goal programming (Lee, 1972) and gradient method of feasible directions (Zoutendijk, 1976). As, in the problem at hand, the objective function is linear and only two constraints are nonlinear, the procedure is simplified when compared with general convex programming problems.

Iterative proceedings are done in five steps with an initial step being used only at the beginning i.e. in initial iteration.

The initial step sets all solution vector values to zero and uses this point as origin x_o in which all nonlinear portions gradient values of the goals are equal to zero. By this the problem of NGP (of (1) to (4)) is reduced to linear approximation solvable by the modified simplex method of linear goal programming.

The first step of an NGP algorithm is the computation of gradients of all constraints together with checking the status of nonlinear constraints in order to deflect, if need be, the gradient of active nonlinear constraints and to avoid zigzagging (see: Zoutendijk 1976, and Ritter 1975). Without this deflecting, the algorithm may converge upon a suboptimal point (see: Van de Panne and Popp, 1963, pp. 411—418).

In the second step the formulated linear goal programming problem is solved by modified simplex method. Namely, nonlinear constraints are transformed to linear on the basis of Euler's theorem computed gradient value in point x_p (in the initial step it was x_o). The modified simplex method of LGP derives the feasible solution x'_p. In concordance with Euler's theorem conditions it is noted that if $f(x_1, x_2, \ldots, x_r)$ is positively homogeneous of degree $r\,(r > 0)$ and the first-order partial derivates exist, then it can be shown that is:

$$f(x_1, x_2, \ldots, x_n) = \frac{1}{r} \sum_{i=1}^{n} \left. \frac{\partial f(x_1, x_2, \ldots x_n)}{\partial x_i} \right|_{x_p} \cdot x_i$$

where, for Cobb—Douglas's functions (5):

$$\left. \frac{\partial f(x_1, x_2, \ldots, x_n)}{\partial x_k} \right|_{x_p} = h\, b_k x_k{}^{(b_k - 1)} \left(\prod_{\substack{i=1 \\ i \neq k}}^{n} x_i b_i \right) \quad \forall\, x_i \geq 0$$

are the coefficients of linearized constraints calculated in point x_p.

The third step serves for feasible direction $s_p(x)$ computation:

$$s_p(x) = x'_p - x_p$$

which has to be "searched" according to the constraints and structure of priority in order to improve the simplex solution.

In the fourth step, the optimal solution vector movement size (step size) is determined by linear searching in the feasible direction identified in the previous step. This way, first the structural constraints A_i (x) must be satisfied and then the goal ones, starting from the goal constraint G_i (x) in (2) which contains the deviation variable with top priority in the objective function of the NGP problem.

To perform the search and to prevent infinite moving in the cases of unbounded problems, it is necessary to determine the lower and upper movement bounds in feasible direction and unit movement size (increment) within these bounds. It is clear that convergence of problem at hand and speed of convergence and other algorithm features depend on the choice of vector s_p (x) and bounds for the step size 1_p.

Initially, the lower limit of 1_p in the algorithm was set to 0 and the upper to 1, but it is possible to define them differently if a need be. The unit movement size in the algorithm at first was set on 0.1, but it is possible to increase the search density.

By these procedures we define whether the first goal constraint (per priority and not per order) is satisfied within the initial bounds. If it is satisfied at some smaller interval $[1_{pl}, 1_{pd}]$, for all $1_{pl} \geq 0$ and $1_{pd} \leq 1$, the search for a further goals constraint goes on exclusively within this interval as its satisfaction, in accordance with so-called Pareto optimality, cannot be sought to the detriment of satisfaction of a higher priority goal.

If during the search we come across a constraint which has not been satisfied previously by goal constraint set bounds, then within these bounds the algorithm identifies the value of movement size for which the deviation from the subject constraint is the least 1_p, we complete the searching and compute the new solution (successor) as follows:

$$x_{p+1} = x_p + 1_p s_p (x_p)$$

If all constraints are satisfied then the new solution is at the same time the optimal solution to a set problem.

In the fifth algorithm step we check the problem convergence by previously set small value of the desired level of convergence accuracy—ε:

$$| (x_{p+1} - x_p) | \leq \varepsilon \ \forall\ p = 1, 2, \ldots,$$

which means that the problem has converged and the algorithm completes its work. Otherwise, the procedure is repeated starting from the first step.

The interactive work of MINGP is supported by the algorithm-included subsystem for establishing the priority structure — PRIOR and subsystem POST for solution improvement on the basis of post-optimal analysis and priority structure change.

A decision maker can but need not be in a position to formulate the priority structure precisely i.e. to define the objective function or to outrank the goals according to their meaning. If the decision maker is not in such a position, the subsystem for establishing the priority structure offers the user a list of possible goals asking the following questions:

— could you outrank the goals, at least partially,

— could you group the goals,

— could you select the most important/the least important group of goals,

— do'you want to assign different priorities within this group, in an attempt to obtain at least partially a differentiated priority structure.

This subsystem also gives the possibility of assigning different weights to goals of the same priority group whether the weights are determined by the decision maker alone or by a program on the basis of random choice.

On the basis of priority structure change the POST subsystem, if so requested by a decision maker, starts up and carries out the postoptimal solution analysis and/or keeps searching for a better solution. The work of this subsystem goes on in the next few steps:

1. analyses of the results obtained by algorithm NGP in order to determine the level of satisfaction of certain priorities in objective function,
2. diagnosing the conflict within the priority structure and establishing the method for its solution,
3. memorizing the best former solution,
4. priority structure change in objective function according to the information obtained in steps 1 and 2,
5. resetting the algorithm NGP, this time from the point of the former best solution with the priority structure defined in step 4,
6. new solution finding and interaction with user to check whether he or she is satisfied with this solution or not,
7. proclamation of the new solution for the former best solution, if user is satisfied and, otherwise, resetting the memorized former best solution to the operative computer memory,
8. examination of further possibilities for solution improvement and, if there are some, returning to step 3 or, otherwise, going to step 4,
9. printing final results.

By the approach "keep finding out better solution" the initial problem solving time is significantly decreased in the case when a decision maker finds, through insight into the former best solution, an often unforeseeable dependence between priorities i.e. goals. In that case it is not necessary to change the priority structure and to restart the NGP algorithm from the initial step. The subsystem for solution improvement includes the NGP algorithm from the point of the former best solution and in many ways decreases the necessary number of iterations.

Computer programs for the methodology of interactive nonlinear goal programming MINGP were coded in Fortran IV Plus on a VAX—11/780 computer and in Fortran 77 on a IBM compatible personal computer.

4. NONLINEAR GOAL PROGRAMMING MODEL IN OPTIMAL ECONOMY DEVELOPMENT POLICY CHOOSING

The standard approach to the solution and simulation of econometric model is well known and often applied in our field whether it relates to Tinbergen's fixed goal approach, Theil's flexible goal approach or the one based on optimal control theory assumptions. In this research MINGP represents the attempt to set the system basis for decision making support in the domain of economic policy optimization i.e. in the condition of model nonlinearity, solution to the problems with more mutually conflicting goals and a partially differentiated decision maker's preferential structure.

MINGP has been tested on a new model of optimal economy development policy choosing based on a goal programming model, the LAMUR model, which differs from one applied up to now in its approach and problem solution methods.

This optimization model enables solutions to the development policy formulation problem and, together with MINGP- optimal decision making for the national economy in which it successfully replaces the simultaneous equations model.

The LAMUR optimization model is based on an econometric model which can be described as: a productional, nonlinear, aggregate, macroeconomic, dynamic simultaneous equations model of smaller dimensions. The main equation in this model is the production function, which starts and ends the cruise flow of an open economy economic activity.

Thus the starting hypothesis in this research was that the essence of economic disturbances (breakdowns, dissonance) in the Yugoslav economy was more due to insufficient supply of goods, while excessive demand was only a secondary occurrence (see: Dujsić, 1988).

The model presented here provides a better understanding of the stochastic aspects of a developing economy by examining the changes that occur over time.

In order to have a better insight into the system of relationships and relations, we could present our model roughly by seven blocks:

B1 = block of production,
B2 = block of prices,
B3 = block of investments,
B4 = block of self consumptions,
B5 = block of personal incomes,
B6 = block of foreign exchanges,
B7 = block of fiscal revenues.

The blocks are mutually connected by some variables (see Figure 1).

As is known, economic policy measures in econometric models are presented by instrumental variables. Therefore the solution to the problem of economic policy choice in such models has been reduced to determining the instrumental variable values.

The equations of the econometric model are estimated by the ordinary least squares method along with the F-test for the test of significance of the estimated coefficients and regression validation. The autocorrelation has been tested by a Durbin—Watson statistic defined by 5% points. The presence of autocorrelation is rejected by the Cochrane—Orcutt procedure.

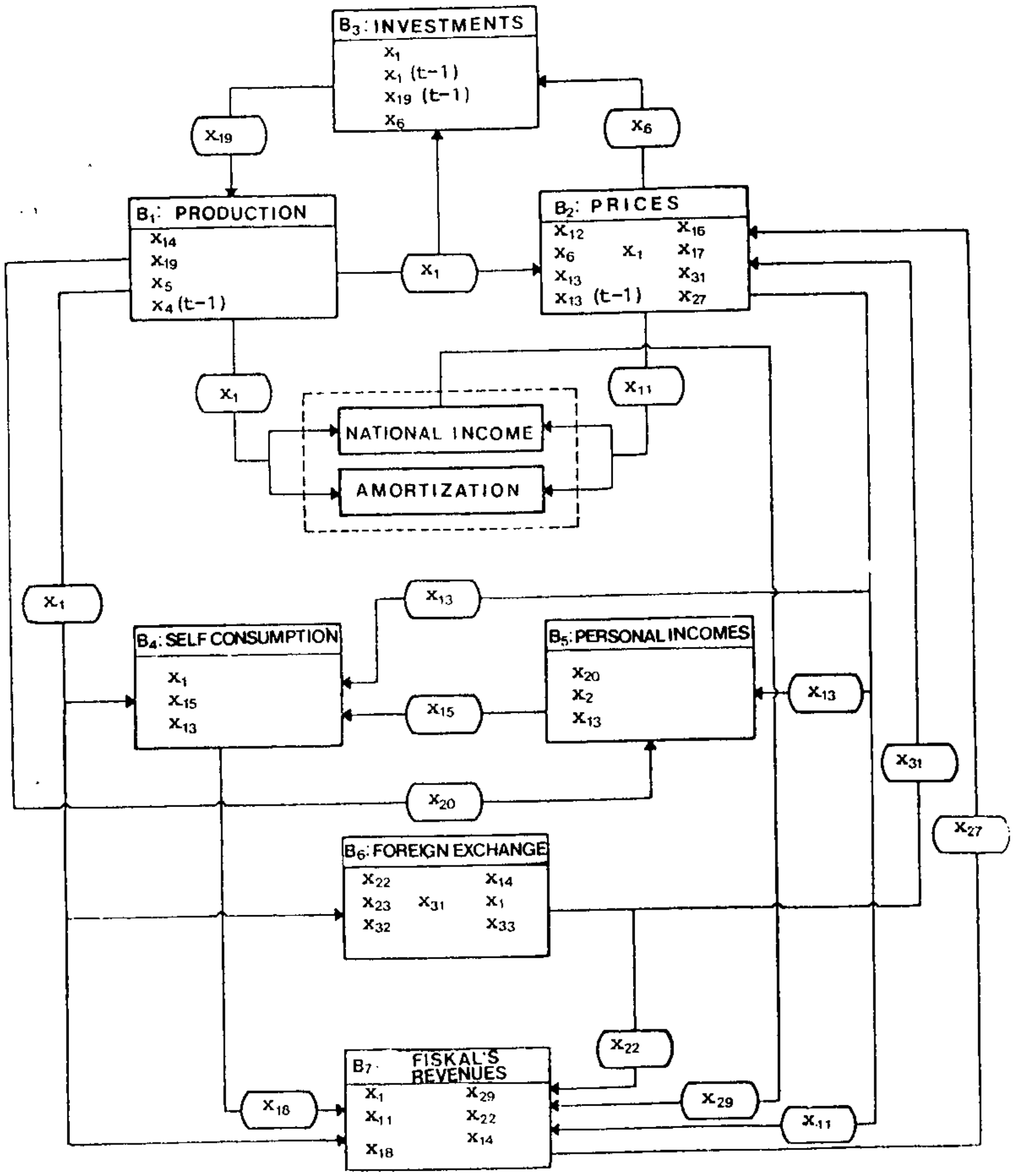

Figure 1. *Block diagram of the LAMUR econometric model*

For econometric model validation for forecasting and consistency evaluation, by ex post simulation we have employed constant term adjustment (see: Klein and Young, 1981).

The kernel of the econometric model is one sort of combination of demand and supply models. The model consists of 27 equations 21 of which are stochastic and 6 are defining.

Since the values of the model variable were measured in different measures, before MINGP application, we ensured the "normalization" of some of the equations by reducing the multidimensional feasible solution area on the dimensions scale between 0 and 20.

Once the statistical tests on the data were satisfactorily completed, input parameters estimated, and several ex post tests performed to validate the model and to estimate the minimal relative size of estimation error, the LAMUR model was run for predicting optimal economic policy during the next one year planning period.

Dynamically, we first applied MINGP as a test of consistency of the planned goals for the next year. In the planning documents the main quantitative economic goals were:

— increase in social product by 3%,
— increase in inflation by 45% and
— increase in gross investments by 2%,

which in essence are conflicting goals (see: Bajt, 1979, pp. 425—427).

The dynamic simulation of LAMUR, solved by MINGP as a single objective nonlinear programming problem, showed that such a goal constellation of Yugoslav economic policy would not be realised without destroying the structural constraints of the existing econometric model.

Therefore the input file of the optimization policy choice problem, consists of the following relaxed goals:

— increase in social product by a rate greater than 2.2%,
— increase in inflation by a rate greater than 43% but less than 88% and
— increase in gross investments by 2%.

The mathematical model formulation of the choice of optimal economy development policy, constructed via a combination of dynamic simulation and optimization techniques, can then be formulated as a nonlinear goal programming problem as below:

Minimize

$$P_1 (\overset{+}{d}_{39} + \overset{-}{d}_{40} + \overset{-}{d}_{43} + \overset{-}{d}_{44}) +$$

$$P_2 (\overset{+}{d}_2 + \overset{-}{d}_2 + \overset{+}{d}_{24} + \overset{-}{d}_{24} + \overset{-}{d}_{25} + \overset{-}{d}_{25}) +$$

$$P_3 (\overset{+}{d}_{28} + \overset{-}{d}_{28}) + P_4 (\overset{-}{d}_{29} + \overset{-}{d}_{29}) +$$

$$P_5 (\overset{+}{d}_{30} + \overset{-}{d}_{30}) + P_6 (\overset{-}{d}_{31} + \overset{-}{d}_{31}) + P_7 \overset{-}{d}_{46}$$

subject to the constraints from (1) to (46):

1) $100X_1 - 100X_2 - 10X_3 = 0.0$

2) $100X_2 - 155.27821X_4^{0\ 538509}\ X_5^{0.461491} + \overset{-}{d}_2 - \overset{-}{d}_2 = 0.0$

3) $10X_3 - 8.12X_2 = 24.351247$

4) $1000X_4 - 5.92362X_{19} - 975.69X_{34} = 0.210555$

5) $X_7 - X_5 - X_8 = 0.0$

6) $X_8 - 0.0021X_3 - 0.9524X_{35} = 0.002298$

7) $X_9 - 0.061X_1 - 0.15327X_7 = 0.007853$

8) $X_{10} - X_7 - X_9 = 0.0$

9) $X_{20} - X_5 - X_9$

10) $10X_{11} - 2.22463X_6 - 1.16842X_{12} - 8.31871X_{13} = 0.0$

11) $10X_{12} - 2.25X_{16} - 6.69X_{17} - 0.27215X_{31} = 0.037344$

12) $10X_6 - 5.41342X_{12} + 0.5382X_{14} = 0.747693$

13) $10X_{13} - 9.46445X_{12} - 0.1047X_{27} - 4.80405X_{36} = 0.6783085$

14) $-1000X_{18} + 12.1346X_1 + 604.3191X_{15} + 1399.81X_{13} = 0.4591351$

15) $10X_{19} - 59.3205X_1 - 2.45899X_6 - 9.91684X_{37} + = 417.229687$

$+ 62.7457X_{38} = 7.09946$

16) $100X_{15} + 7.27287X_{20} - 13.131X_2 - 88.6279X_{13} = 4.813987$

17) $-100X_{21} + 40.792X_{22} + 10.609X_{23} + 38.01588X_{32} = 20.114828$

18) $-100X_{22} + 54.817X_1 - 38.716X_{14} + 76.22366X_{31} = 84.655504$

19) $1000X_{25} - 1000X_{24} + 1000X_{26} + 100X_{27} + 100X_{28} = 0.0$

20) $1000X_{24} - 1276.2744X_{11} - 87.9184X_1 = 284.451904$

21) $1000X_{26} - 186.97X_{29} = 54.436483$

22) $100X_{27} - 131.354X_{18} = 3.951925$

23) $100X_{28} - 9.37X_{22} - 74.7098X_{14} = 12.79104$

24) $1000X_{29} + 1000X_{30} - 1000X_1 X_{11} + \overset{-}{d}_{24} - \overset{+}{d}_{24} = 0.00022$

25) $-1000X_{30} + 105.43X_1 X_{11} + \overset{-}{d}_{25} - \overset{+}{d}_{25} = 25.303613$

26) $-X_{31} + 2.1896X_{33} = 0.65596$

27) $-X_{32} + 1.8761X_{33} = 0.71605$

28) $2.86816X_{14} - 3.556X_{11} + \overset{-}{d}_{28} - \overset{+}{d}_{28} = 0.0$

29) $2.86816X_{16} - 1.8635X_{11} + \overset{-}{d}_{29} - \overset{+}{d}_{29} = 0.0$

30) $2.86816X_{17} - 2.3651X_{11} + \overset{-}{d}_{30} - \overset{+}{d}_{30} = 0.0$

31) $2.86816X_{23} - 3.353X_{11} + \overset{-}{d}_{31} - \overset{+}{d}_{31} = 0.0$

32) $X_{10} \leq 7.7$

33) $X_{33} = 2.1$

34) $X_{34} = 1.01952$

35) $X_{35} = 0.138$

36) $X_{36} = 2.89485$

37) $X_{37} = 8.06245$

38) $X_{38} = 3.93641$

39) $X_1 + \overset{-}{d}_{39} - \overset{+}{d}_{39} = 4.0221$

40) $X_1 + \overset{-}{d}_{40} - \overset{+}{d}_{40} = 5.5$

41) $X_5 \geq 5.40294$

42) $X_5 \leq 5.5$

43) $X_{11} + \overset{-}{d}_{43} - \overset{+}{d}_{43} = 5.4$

44) $X_{11} + \overset{-}{d}_{44} - \overset{+}{d}_{44} = 4.1$

45) X_{14} ≥ 5.334

46) $X_{19} + \overset{-}{d}_{46} - \overset{+}{d}_{46}$ $= 8.22327$

In the first 27 terms the equations of the econometric model have been given. In terms from 28th to 31st row the functions of instrumental variable value are given. The term in 32nd row contains the upper bound of available working people (in millions). In the 33rd row the time variable assumes that first year ($X_{33} = 0.1$) is 1966th, and rows from 34th to 38th give the value for time lagged variables $X_{4(-1)}$, $X_{8(-1}$, $X_{13(-1)}$, $X_{19(-1)}$ and $X_{1(-1)}$ respectively. The terms from 41st to 46th row contain the bounds values for instrumental variables.

The first priority in the initial objective function is given to satisfying the goal of social product growth of both sectors (of variable X_1), and to keeping the inflation rate (X_{11}) in previously given bounds. The second priority goal is satisfying of all nonlinear functions in the model.

In the 3rd, 4th, 5th and 6th priority in the objective function are the requirements for minimization of deviations from goal instruments of economic policy and in the 7th priority is the goal requirement for national gross investment growth.

5. RESULTS

This problem is solved after the 3rd interactive trial and 15 iterations of the NGP algorithm to detremine the best solution. The MINGP aplication to this model yields the following optimal solution:

$X_1 = 4.02210$	(Social product of both social and individual sectors at CP)
$X_2 = 3.49481$	(Social product of soc. sector at const. prices-CP)
$X_3 = 5.27291$	(Social product of individual sector at CP)
$X_4 = 1.04443$	(Basic means of production purchase value of social sector at CP)
$X_5 = 5.5$	(Number of workers in social sector of economy)
$X_6 = 1.98261$	(Index of growth of investment goods at CP)
$X_7 = 5.64480$	(Number of workers in both soc. and indiv. sec.)
$X_8 = 0.14480$	(Number of workers in individual sector)
$X_9 = 1.11838$	(Number of workers in nonproductive sector)
$X_{10} = 6.76318$	(Total number of workers in both productive and non-productive sector)
$X_{11} = 5.4$	(Implicit deflator of social product at CP)
$X_{12} = 4.06739$	(Producer's price index at CP)
$X_{13} = 5.42617$	(Costs of living index at CP)
$X_{14} = 5.33400$	(Average exchange rate of dinar at import)
$X_{15} = 4.83480$	(Net incomes of both productive and nonproductive sector of social sector)
$X_{16} = 3.50849$	(Monetary mass)
$X_{17} = 4.62770$	(Stock growth increase)
$X_{18} = 10.14894$	(Self consumption)

$X_{19} = 8.352890$ (Gross investment in basic funds)
$X_{20} = 6.618380$ (Employed people in social sector in both productive and nonproductive sectors)
$X_{21} = 2.631530$ (Export of goods and services at CP)
$X_{22} = 2.298020$ (Import of goods and services at CP)
$X_{23} = 6.312830$ (Average exchange rate of dinar at export)
$X_{24} = 7.529950$ (Revenues of fiscal system)
$X_{25} = 2.068170$ (Other revenues of fiscal system)
$X_{26} = 3.691900$ (Taxes, fees and contributions, excluding sales taxes)
$X_{27} = 13.37056$ (Sales tax)
$X_{28} = 4.328260$ (Customs revenues)
$X_{29} = 19.45477$ (National income)
$X_{30} = 2.264570$ (Amortization)
$X_{31} = 3.942200$ (Index of gross import prices at CP)
$X_{32} = 3.223760$ (Index of gross export prices at CP)

This solution has been obtained in the 3rd interactive trial, after the two previous interactions have not given a satisfactory solution for achievement of the 2nd and 3rd goals, but in such a way that the 3rd goal in the initial objective function was renamed as the first and the 2nd as the 3rd, while the order of other priorities remained unchanged.

This fact points to the importance and potential role of the approach offered by MINGP in relation to ad hoc, often pragmatic approaches. Namely, it must be totally clear that a decision maker could not, by a simple deductive procedure, reach such a conclusion as the improvement of satisfying the 2nd goal could be accomplished indirectly by renaming the 3rd goal as the first one.

In essence, mathematically speaking, MINGP has found three local minimums among which, in collaboration with the decision maker, it has selected the most satisfactory solution.

Otherwise, the optimum solution of economic policy for the planned year, in respect to the solution by dynamic simulation, has revealed that there were possibilities for a further dynamizing of production activities in the Yugoslav economy but not with such intensity of social product and inflation growth as was predicted. In other words, it has been shown that the Yugoslav economy, wishing to stabilize, must look for a trade-off between growth and stabilizing under limited investments (this being the problem of most developing countries) at the expense of social product growth. In addition to the solution of instrumental variables of monetary and fiscal policy the specific solution given in this paper demonstrates that trade-off is an economic policy which would give a social product growth of 2.2% and inflation growth of 54% (not 45%), in other words, inflation of 45% cannot be maintained at a social product growth of 3% but at a level almost double.

6. CONCLUSIONS

The purpose of this research and its applicative part was to construct a new structure for an econometric model of the Yugoslav economy, to present its application in analyses of economic activities and to enable the projection of an economic policy variant which would bring about an optimum harmony of production, consumption and inflation. This harmony represents in fact the model of a stabilization policy which can be carried out by the measures and instruments of economic policy.

There is great scope for further research by multidisciplinary research workers equipping themselves for better familiarization through the formation of realistic systems and more reliable model approximation. This would bring economic policy into a situation of stable trends instead of ad hoc and suboptimal appraisal.

By further extension of MINGP the assumption could be created for overcoming the current limitations in econometric problem solving, in which the stochastic equations in the solution procedure are treated as determining statements.

There are no practical limitations to the application of MINGP and econometric models to some lower organizational forms (regions, enterprises etc.) if an adequate data base is provided i.e. reliability and availability of appropriate statistical-documental bases.

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PRIMJENA MODELA NELINEARNOG CILJNOG PROGRAMIRANJA PRI IZBORU OPTIMALNE POLITIKE PRIVREDNOG RAZVOJA

Lazo ROLJIĆ & Muris DUJSIĆ

U ovom radu na bazi dosad poznatih metoda razvijen je novi model nelinearnog ciljnog programiranja — NCP i nova metodologija interaktivnog nelinearnog ciljnog programiranja — MINCP. Osnovu MINCPa čine metoda linearnog ciljnog programiranja i gradijentna metoda dopustivih smjerova. Koristeći Euler-ovu teoremu o homogenosti funkcije, MINCP vrši linearizaciju nelinearnih homogenih funkcija ograničenja u problemu nelinearnog ciljnog programiranja, pa problem svodi na linearni kod kojeg su tehnološki koeficijenti uz vari-

jable odlučivanja gradijenti izračunati u okolini neke tačke na dopustivom smjeru. Interaktivni rad MINCP-a odvija se u vidu jednostavnih pitanja i odgovora putem podsistema PRIOR — kod formulisanja funkcije kriterija i podsistema POST — kod analize Pareto optimalnih rješenja i ponovne formulacije funkcije kriterija izmjenom prioriteta ili teženiskih koeficijenata za pojedini prioritet. U empirijskom dijelu rada primjena MINCP-a bavi se potpuno novim pristupom modeliranju optimalne makroekonomske politike. Jezgro optimizacionog modela je ekonometrijski model dobiven analizom kauzalnih odnosa u procesu društvene reprodukcije u SFRJ u poslednjih 20 godina. Ekonometrijski model sastoji se iz blokova: proizvodnje, lične i investicione potrošnje, cijena, ličnih dohodaka, razmjena sa inostranstvom i fiskalnih prihoda. Osnovna jednačina modela je Cobb—Douglas-ova proizvodna funkcija. Problem optimalne makroekonomske politike formulisan je kao problem nelinearnog ciljnog programiranja. Prvi prioritet u funkciji kriterija dat je rastu ukupnog društvenog proizvoda i smanjenju inflacije. Ex-ante simulacija i optimizacija ekonomske politike za 1986. godinu pokazali su da je u SFRJ bila rezolutna nekonzistentna makroekonomska politika koja je vodila usporavanju privrednog razvoja i porastu inflacije. U radu su prikazani odgovarajući algoritmi, izgled optimizacionog modela i numerička rješenja postavljenog problema izbora optimalne politike privrednog razvoja.