ECONOMIC ANALYSIS AND WORKERS'
MANAGEMENT, 2, XXV (1991), 179—186

A STOCHASTIC LINEAR OPTIMIZATION MODEL

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ABSTRACT

An optimization model [1], [2] for process production, which is representatively described by a finite number of random variables, is obtained by using factor analysis [3] and linear multiple regression. The interdependences of endogenous random variables are also built into the model. The model and the corresponding computer program SLOM are permanently applied for ferrosilicon production at the factory Tovarna dušika Ruše.

1. INTRODUCTION

The result of a process production depends upon a set of dependent random variables. For the application of this model a random sample for all random variables, which describe the process, is needed. All random variables have to be normally distributed, what is not a very strong supposition, because the time random sample is obtained by measurements under equal conditions.

2. CONSTRUCTION OF THE MODEL

Let

$$y_j \qquad j = 1, 2, \ldots, n \tag{1}$$

be normally distributed random variables, with which the considered process can be representatively described and let

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$$y_N = [y_{jk}] \qquad j = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, N \tag{2}$$

be the corresponding random sample. The standardized random variables are denoted by

$$x_j \qquad j = 1, 2, \ldots, n \tag{3}$$

By using factor analysis, the standardized random variables can be expressed by a system of linear equations

$$x_j = \sum_{k=1}^{m} q_{jk} f_k + e_j \qquad j = 1, 2, \ldots, n; \quad m < n \tag{4}$$

where f_k are common factors, e_j residuals and q_{jk} factor loadings. The number of endogenous variables will be determined by the number of common factors, since in the case of independence of endogenous random variables there exist m random variables from (3), which can be considered as a solution of the system (4).

Let us denote by r_{ij} $(i, j = 1, 2, \ldots, n)$ the sample correlation coefficients of (3). The endogenous variables are chosen as follows. The variable x_1 is determined arbitrarily. The variable x_2 is chosen such that r_{1j}^2 $(j = 2, 3, \ldots, n)$ is minimal. Generally, x_{k+1} is chosen such that

$$\sum_{i=1}^{k} r_{ij}^2 \longrightarrow min \qquad j = k + 1, \ldots, n$$

Let us assume for the sake of simplicity that

$$x_1, x_2, \ldots, x_m \tag{5}$$

are endogenous random variables determined as described above. Linear multiple regression supposes independence of endogenous variables, which rarely occurs in applications. So the influence of the dependence of endogenous variables is built into the model.

Because of the normal distribution of variables (5) the dependence between the endogenous variables x_j and x_i can be expressed by using linear regression

$$x_j = r_{ji} x_i + e_{ji} \qquad i, j = 1, 2, \ldots, m; \; i \neq j \tag{6}$$

let

$$\tilde{x}_j \qquad j = 1, 2, \ldots, m \tag{7}$$

be continuous deterministic variables which, in case of independent random variables (4), define variable means of the variables (5). Initial value of each variable (7) equals 0 because of the standardization of the values of the variables (7) can also be negative, then the means can increase or decrease. Since the variables (5) are rarely independent, by equations

$$x_{j(i)} = r_{ji}\tilde{x}_i \qquad i, j = 1, 2, \ldots, m \tag{8}$$

the influence, caused by the variation of the random variable x_i ($i = 1, 2, \ldots, m$) mean on the variation of the random variable x_j ($j = 1, 2, \ldots, m$) mean, is expressed. The equations (8) are fullfiled also in case $i = j$, since $r_{ii} = 1$ and $x_{i(i)} = \tilde{x}_i$.

The sum of all influences (8) expresses the influence of means changes of all variables x_i ($i = 1, 2, \ldots, m$) on the mean change of the variable x_j ($j = 1, 2, \ldots, m$).

$$x_{j(en)} = \sum_{i=1}^{m} r_{ji}\tilde{x}_i \qquad j = 1, 2, \ldots, m \tag{9}$$

Using linear multiple regression, one can express the exogenous random variables by endogenous random variables

$$x_j = \sum_{k=1}^{m} a_{jk}x_k + e'_j \qquad j = m+1, m+2, \ldots, n \tag{10}$$

where e'_j, ($j = m+1, m+2, \ldots, n$) denote residuals.

The means changes of exogenous random variables we obtain by

$$x_{j(ex)} = \sum_{k=1}^{m} a_{jk} \sum_{i=1}^{m} r_{ki}\tilde{x}_i \qquad j = m+1, m+2, \ldots, n \tag{11}$$

Using (9) and (11), we obtain the optimization model in which only deterministic variables occcur:

$$\max \sum_{j=1}^{m} c_j\tilde{x}_j \tag{12}$$

subject to direct conditions

$$\sum_{i=1}^{m} c_{ji}\tilde{x}_i \leqslant d_j \qquad j = 1, 2, \ldots, n \tag{13}$$

and statistical conditions

$$-t_j \leqslant \sum_{i=1}^{m} r_{ji} \tilde{x}_i \leqslant t_j \qquad j = 1, 2, \ldots, m \tag{14}$$

$$-t_j \leqslant \sum_{k=1}^{m} a_{jk} \sum_{i=1}^{m} r_{ki} \tilde{x}_i \leqslant t_j \qquad j = m+1,\ m+2, \ldots, n \tag{15}$$

where t_j $(j = 1, 2, \ldots, n)$ are experimentally determined parameters which mean parts of standard deviations of the variables (1). These parameters determine statistically admissible intervals of means changes of these variables. From the viewpoint of optimization it would be reasonable to choose high values of parameters t_j, but from the point of statistical view this is not admissible, because the dependences (9) — (11) are fulfilled only on the sample area. From our practical experiences we suggest choosing the values of paramerets t_j about 0.5. It is evident that nonnegativity constraints do not occur in the model.

Through the SLOM software package we obtain the optimal values

$\tilde{x}_j^*$ $(j = 1, 2, \ldots, m)$ of the variables (7), the optimal values

$\overline{x}_j^*$ $(j = 1, 2, \ldots, n)$ of means of the variables (3), the optimal values

$\overline{y}_j^*$ $(j = 1, 2, \ldots, n)$ of means of the variables (1) and the maximal value of the objective function.

3. APPLICATION OF THE MODEL

The described model is being applied for ferrosilicon production at the factory Tovarna dušika Ruše. Ferrosilicon FeSi 75 is an 15:75 alloy of iron and silicon, used as deoxidant in ironworks and smelting plants. It is a product of carbothermic reduction of silicon dioxide from quartz according to the equations

$$SiO_2 + 2C \rightarrow Si + 2CO$$

$$Fe + Si \rightarrow FeSi$$

Iron is added as a smelting element. The carbon holders for quartz reduction are coke, coal and lignite. All the thermodynamic reactions are energetically negative, which requires the addition of electrical energy into the furnace (8300—9500 KWh per ton of product). The model for the optimization of ferrosilicon production is applied on an 24 MW submerged arc furnace. It operates with 23 random variables (1), by which the optimized process is defined. The random sample (2) is obtained by measurements every 8 hours. As the values of random variables are the result of measurements in approximately equal conditions, these variables are approximately normally distributed. The list of random variables is given in the table 1.

The most important optimization criterion is the specific consumption of electric energy, because its cost represents cca. 65% of all variable costs and because the minimization of energy consumption is important from the point of view of the national economy. The production quantity, the Si recovery, the SO_2 emission and the direct unit costs, are also important in ferrosilicon production.

j	Y(j)	RANDOM VARIABLES	UNITS
1	Y(1)	active power	MW
2	Y(2)	reactive power	MVar
3	Y(3)	shutdown	hours/8 hours
4	Y(4)	production rate of FeSi	Kg
5	Y(5)	power factor (cosinus φ)	
6	Y(6)	ohmic resistance	mOhm
7	Y(7)	inductive resistance	mOhm
8	Y(8)	specific electric consumption	KWh/t
9	Y(9)	quartz consumption	Kg/8 hours
10	Y(10)	iron scrap consumption	Kg/8 hours
11	Y(11)	black coal consumption	Kg/8 hours
12	Y(12)	lignite consumption	Kg/8 hours
13	Y(13)	coke consumption	Kg/8 hours
14	Y(14)	electrode paste consumption	Kg/8 hours
15	Y(15)	silicon recovery	%
16	Y(16)	% of silicon in the product	
17	Y(17)	% of aluminium in the product	
18	Y(18)	% of calcium in the product	
19	Y(19)	electrode position	cm
20	Y(20)	iron lances consumption	pieces/8 hours
21	Y(21)	taping type (1 = burns, 2 = blowes, 3 = intensive blowes)	1, 2, 3
22	Y(22)	SO_2 emission	Kg/8 hours
23	Y(23)	direct unit costs	m.u./t

Table 1. — *The list of random variables*

Let us denote

$$\tilde{x}^T = [\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_m]$$

and let X be the set of all feasible solutions of the problem (12) — (15).

Five optimization problems

$$\max_{\tilde{x}\in X} f_1(\tilde{x}),\ \min_{\tilde{x}\in X} f_2(\tilde{x}),\ \max_{\tilde{x}\in X} f_3(\tilde{x}),\ \min_{\tilde{x}\in X} f_4(\tilde{x}),\ \min_{\tilde{x}\in X} f_5(\tilde{x})$$

with

$$f_1(\tilde{x}) = \overline{y_4}(\tilde{x})$$
$$f_2(\tilde{x}) = \overline{y_8}(\tilde{x})$$
$$f_3(\tilde{x}) = \overline{y_{15}}(\tilde{x})$$
$$f_4(\tilde{x}) = \overline{y_{22}}(\tilde{x})$$
$$f_5(\tilde{x}) = \overline{y_{23}}(\tilde{x})$$

where $\overline{y_j}(\tilde{x})$ means the changeable mean of the random variable y_j, were solved. For the sake of comparability of optimization results for each parameter t_j the same value $t_j = 0.5$ was taken.

The results of these optimizations are given in the tables 2 and 3. In the table 2 the values of objective functions are given in original units and in table 3 are given percentual changes of all optimization criteria.

	$\max_{\tilde{x}\in X} f_1(\tilde{x})$	$\min_{\tilde{x}\in X} f_2(\tilde{x})$	$\max_{\tilde{x}\in X} f_3(\tilde{x})$	$\min_{\tilde{x}\in X} f_4(\tilde{x})$	$\min_{\tilde{x}\in X} f_5(\tilde{x})$
f_1	18650	18538	17815	17596	18627
f_2	8578.5	8578.5	8960	8795	8578.5
f_3	87	87.4	95	95	86.8
f_4	337	335	333	324.4	336
f_5	1.161	1.161	1.208	1.194	1.158

Table 2. — *Results of optimization in original units*

	$\max_{\tilde{x}\in X} f_1(\tilde{x})$	$\min_{\tilde{x}\in X} f_2(\tilde{x})$	$\max_{\tilde{x}\in X} f_3(\tilde{x})$	$\min_{\tilde{x}\in X} f_4(\tilde{x})$	$\min_{\tilde{x}\in X} f_5(\tilde{x})$
f_1	3.999	3.377	—0.657	—1.880	3.871
f_2	—4.036	—4.036	0.234	—1.613	—4.036
f_3	—3.077	—2.621	5.802	5.802	—3.315
f_4	0.609	0.117	—0.497	—3.094	0.289
f_5	—3.797	—3.799	0.134	—1.026	—4.036

Table 3. — *Percentual changes of optimization criteria*

Each of the chosen optimization criteria is important for managing ferrosilicon production according to ecological restrictions. We can see that the results of the optimization with the criteria f_1, f_2, f_5 are not essentially in conflict, but all of them come into conflict with the criteria f_3 and f_4. It is clear that it is necessary to apply the multicriteria optimization. But for the moment it was decided to manage the process according to the criterion f_5, because the result of the optimization with this criterion shows acceptable changes for the remaining criteria, which are most important for the managers at the factory TD Ruše.

CONCLUSION

The ferrosilicon production process is managed by this model. For the first optimization the random sample was chosen and the process was managed by using optimal solution of this optimization and after 2 months a new sample was prepared. The process was then optimized once again. This procedure was repeated until the optimal value of the objective function changed significantly. The model also offers the expressions of linear dependences of random variables (1). This gives the possibility of variables selection. Some variables do not significantly influence the changes of the objective function.

The described model can be successfully used in agriculture, medicine, biology, pharmacy, metallurgy, in the food, timber, paper, chemical and textile industries and similar.

Received: 10. 07. 1990
Revised: 7. 01. 1991

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STOHASTIČNI LINEARNI OPTIMIZACIJSKI MODEL

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P o v z e t e k

V članku je opisan stohastični model optimizacije procesne proizvodnje. Pogoj za uporabo tega modela je reprezentativni opis procesa s končnim številom normalno porazdeljenih slučajnih spremenljivk. Model temelji na faktorski analizi, linearni multipli regresiji in linearnem programiranju. V model so vgrajene odvisnosti med baznimi spremenljivkami in njihov vpliv na nebazne spremenljivke. Prikazana je primerjalna analiza rezultatov optimizacije proizvodnje ferosilicija v Tovarni dušika Ruše. Rezultati so dobljeni s paketom računalniških programov SLOM.