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„DIVERGENT EQUILIBRIUM" IN A LABOUR-MANAGED ECONOMY[1]

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ABSTRACT

In this note I demonstrate a potential theoretical problem in achieving general equilibrium in an economy composed of competitive firms that maximize income per worker, under conditions of fixed capital; a problem which suggests that disequilibria in some markets might be a rule rather than exception. This result is in contrast with Dreze's (1976) well-known general equilibrium model of a labor-managed economy in which he argues that competitive equilibria in LM and profit-maximizing economies *in the long run* (with free entry and exit) lead to the same set of allocations, which are also Pareto optimal. The model presented here is not directly comparable to that of Dreze, since I keep the number of firms and households constant and, in contrast to Dreze, I do not introduce a distinction between two types of LM firms (one that pays a fixed rental charge for capital – equivalent to profits, and the one that does not). The result in this note, however, reinforces the conventional view that a number of the behavioral peculiarities of LM firms may have serious repercussions for the behavior of an LM economy in general equilibrium.

1. INTRODUCTION

The basic partial equilibrium model of a labor-managed (LM) firm[3] developed by Ward (1958) has initiated a rapid growth in the literature on the economics of labor-management. Many authors criticized and developed

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[3] Ward (1958), in his seminal paper, named the firm that maximizes income per worker »Illyrian firm«. The term labor-managed firm with an identical meaning has been dominant in the recent literature.

various parts of the Wardian model: Domar (1966), Horvat (1967), Robinson (1966), Vanek (1970) and Meade, *inter alia*. More recently, several authors have analyzed the behavior of a labor-managed firm under conditions of imperfect conditions, uncertainty, as well as in a general equilibrium setting (Dreze, 1976). Bonin and Putterman (1986) and Ireland and Law (1982) offer good reviews of this rapidly growing literature.

Negatively sloped output supply curve, Pareto suboptimality, the unusual direct relationship between the fixed costs on one hand, and employment and output on the other, the inverse relationship between the demand for labor and the price of output are a few of the well known Ward-Vanek paradoxes of a labor-managed firm. Theoretical predictions of the Wardian model have often been discussed in the context of the Yugoslav economy, and have been the subject of heated debates[4], but they still dominate the existing literature.

In this note I demonstrate a potential theoretical problem in achieving general equilibrium in an economy composed of competitive firms that maximize income per worker, under conditions of fixed capital; a problem which suggests that disequilibria in some markets might be a rule rather than exception.

2. THE SHORT-RUN BEHAVIOR OF A LABOR-MANAGED FIRM

A representative LM firm operates in a perfectly competitive environment and combines labor (L) with a fixed amount of capital (K) to produce a final consumption good (Q). Production technology is given by the production function $Q(L, \bar{K})$, where $Q_L > 0$ and $Q_{LL} < 0$. The firm is subject to an *ad valorem* tax (t), and receives an *ad valorem* capital subsidy (s). Thus, the LM firm's problem is to choose L to maximize the income per worker function:

$$Y = \frac{(1-t)PQ(L,\bar{K})-(1-s)r\bar{K}}{L} \tag{1}$$

Where Y is income per worker, P, is the market price of output, and r is the full price of capital.[5]

The necessary condition for maximizing (1) is:

$$(1 - t)\, P\, (LQ_L - Q) + (1 - s)\, r\bar{K} = 0 \tag{2}$$

which, in general, can be solved for the firm's labor demand:

$$L_d = L_d\, (P, r, \bar{K}, t, s, v) \tag{3}$$

where v is the vector of production function parameters. Substituting this expression into the production function and the objective function (1) gives the firm's output supply function:

$$Q_s = Q_s\, (P, r, \bar{K}, t, s, v) \tag{4}$$

and the resulting income per worker function:

$$Y = Y\, (P, r, \bar{K}, t, s, v) \tag{5}$$

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[4] See the exchange between Horvat and Madžar (1986).

[5] The price of capital here means the full cost of capital, or opportunity cost of capital.

Unlike the competitive profit-maximizing firm, which faces a parametric wage or salary for the workers it employs, the income per worker is *endogenously* determined by the LM firms's decision about the number of workers to employ.

Output price (P) clearly plays a key role in determining the behavior of the LM firm. Although each firm in this model regards P as a parameter, output price ultimately will be endogenous in the general equilibrium framework. For simplicity, and to identify their roles in the general equilibrium model, these three behavioral functions of the LM firm will be denoted L_d (P), Q (P), and Y (P), respectively.

The partial equilibrium literature has noted the perverse responses of the LM firm to exogenous changes in P. With minimal restrictions on the firm's technology ($Q_L > 0$, $Q_{LL} < 0$), the nature of this response easily can be seen. Since (2) determines the firm's choice of L, the implicit function rule can be used to obtain:

$$\frac{\delta L_d}{\delta P} = -\frac{(LQ_L - Q)}{PLQ_{LL}} < 0 \tag{6}$$

if $Q_{LL} < 0$ and $(LQ_L - Q) < 0$. The latter condition must hold for the first-order condition to be satisfied, and diminishing marginal productivity of labor ($Q_{LL} < 0$) is a standard assumption. From (6), it also follows that:

$$\frac{\delta Q_s}{\delta P} = Q_L \left(\frac{\delta L_d}{\delta P}\right) < 0 \tag{7}$$

This downward-sloping short-run supply of the LM firm appears to be a rather general result that contrasts sharply with the response of a competitive profit-maximizing firm. Within a general equilibrium setting, such behavior raises questions about the existence of an equilibrium price, multiple equilibria, and price stability problems. However, before the general equilibrium model can be formulated, the economic behavior of the typical household must be considered.

3. THE BEHAVIOR OF A CONSUMER-WORKER HOUSEHOLD

The typical household is annually endowed with T person-years of time that must be allocated to leisure (non-work) activities (Z) and labor (L). The annual income per worker (i.e., per person-year of labor) is Y and is subject to a proportional income tax (t_i). The total post-tax income of the household, $(1 - t_1)$ YL, is spent on a composite consumption good (Q). If both consumption and leisure are valued by the household, the choice problem is to maximize:

$$(Q, Z, \mu) = U(Q, Z) + \mu[(1 - t_i) Y (T - Z) - PQ] \tag{8}$$

where U (Q, Z) is a well-defined utility function, μ is a nonnegative Lagrange multiplier, and P is the price of the composite good.

The necessary conditions for the maximization of (8) are:

$$U_q - \mu P = 0 \tag{9}$$

$$U_z - (1 - t_i) Y = 0 \tag{10}$$

$$(1 - t_i) Y (T - Z) - PQ = 0 \tag{11}$$

The solution of this system of equations gives the household's product demand:

$$Q_d = Q_d (P, Y, T, t_i, w) \quad (12)$$

its demand for leisure,

$$Z_d = Z (P, Y, T, t_i, w) \quad (13)$$

and its marginal utility of income,

$$\mu = \mu (P, Y, T, t_i, w) \quad (14)$$

where w is the vector of utility function parameters. From the time constraint ($T = Z + L$) and (13), we also can obtain the household's labor supply function:

$$L_s = L_s (P, Y, T, t_i, w) \quad (15)$$

In the general equilibrium model that follows, (12) and (15) will be denoted more compactly as $Q_d (P, Y)$ and $L_s (P, Y)$. Within the present partial equilibrium model of the household, both product price and income per worker exert an influence on product demand and labor supply. When this behavior is merged with the behavior of firms in a general equilibrium setting, however, it should be recalled from the previous section that income per worker is dependent on output price in a labor managed economy. Thus we also may write:

$$Q_d [P, Y (P)] = Q_s (P) \quad (16)$$

$$L_s [P, Y (P)] = L_d (P) \quad (17)$$

4. A SIMPLE GE MODEL WITH EXOGENOUS GOVERNMENT POLICY

Consider a simple GE model consisting of Nf LM firms and Nh households. Government policies influence the behavior of each group in an exogenous manner, and without any budgetary limitations. This would be clearly an inappropriate assumption in a model concerned with the optimal mix of government's revenue instruments under a fixed budget constraint. This model, however, has a different focus: analyzing the behavior of a labor-managed economy, and certain problems related to achieving market equilibria.

Using the behavioral functions derived from the partial equilibrium models of the representative firm and household in the previous sections, simultaneous clearance of product and labor markets in a labor-managed economy would require the following two equations to hold:

$$Nh.Q_d (P, Y) - Nf.Q_s (P) = 0 \quad (18)$$

and

$$Nf.L_d (P) - Nh.L_s (P, Y) = 0 \quad (19)$$

where $L_d (P)$ and $Q_s (P)$ are given by equations (3) and (4), and $Q_d (P, Y)$ and $L_s (P, Y)$ are given by equations (12) and (15).

Absent further restrictions, and given specific behavioral functions, one could solve this system for equilibrium values of P and Y. In a labor-managed

economy, however, there exists a derived link between Y and P, given by equation (5). Substituting this expression into the above system gives:

$$Nh.Q_d\,[P, Y\,(P)] - Nf.Q_s\,(P) = 0 \tag{20}$$

$$NF.L_d\,(P) - Nh.L_s\,[P, Y\,(P)] = 0 \tag{21}$$

From (20) and (21), it is apparent that the output price (P) plays a critical role in both the product market and the labor market. The fact that labor market clearance depends on P stems from the endogenous nature of income per worker. The structure of this simple model also points to a fundamental dilemma for a labor-managed economy: **the value of P that clears the product market (20) need not clear the labor market (21), and vice versa.** In such an economy, the fundamental challenge for policymakers may be finding policy combinations that will cause product and labor markets to equilibrate at a common value of P.

5. CONCLUSION

Having demonstrated the nature of »divergent equilibrium« in a labor-managed economy, in a simple Marshallian, restricted general equilibrium framework, the questions of interpretation and empirical significance naturally arise. It is also useful to highlight a few assumptions on which the model rests; this may also suggest avenues of future research.

First, in this extremely simplified world, with a typical LM firm as a price taker, in a one variable input, one-output case, fixed capital, and a representative optimizing consumer-worker household, the model implies the inability of the system to simultaneously clear both product and labor markets: one market will be in disequilibrium. Walras' law does not hold, even in the simplest two-market model. Clearly, the result supports basic inferences often drawn from partial equilibrium Wardian models, that maximization of income per worker leads to behavioral inefficiencies stemming from the *implicite* postulated inverse relationship between income per worker and demand for labor. Moreover, it shows that some of these inefficiencies may well spill over into the general equilibrium setting.

Second, the result may be dependent on the assumptions built in the model, such as fixity of capital, and absence of entry of new firms. However, the purpose of this note was to analyze the behavior of a purely Wardian labor-managed economy, with flexible product price and income per worker (price of labor). The conclusion that LM economy in the short run suffers from the nonexistence problem is, however, not incompatible with Dreze's (1976) famous proposition on the long-run equivalence between the profit-maximizing and the labor-managed economy. It may well be, as often stressed by Vanek, that for an LM economy to function efficiently (i.e., overcome short run disequilibria of the sort described in this note), a strong entry mechanism of new firms must exist.

Third, the postulated Wardian equivalence between adjustment mechanisms in labor-managed and profit-maximizing firms may also be another critical assumption on which the result depends. Namely, it is assumed that LM firms react to changes in their environment by quantity adjustment in

the only variable input (labor). This has been a standard assumption in most partial equilibrium literature on LM firms which relies on original Wardian model. However, it is possible to imagine other adjustment mechanisms[6], which can yield different results. Recently, Whyte and Whyte (1988), and Bradley and Gelb (1986), described Mondragon's unusually flexible pay policy and worker-relocation policy, which were largely responsible for the remarkable ability of the Spanish cooperative complex to survive and grow during a deep recession that hit Spain in late 1970s and early 1980s.

Finally, it is worth noting a recent model by Šuvaković (1988) which rectifies Ward-Vanek paradoxes in the standard partial equilibrium framework, by introducing a regressive taxation scheme in the LM firm's choice problem. Given sufficiently high regressiveness of the tax rate, Šuvaković shows that comparative statics of LM firm and PM firm can be almost identical. Intuitive inference from this result suggests that this would probably solve the nonexistence problem described in an LM economy, regressive taxation scheme could have detrimental effects on the competitiveness of market structures and entry of new firms.

Another line of research is to test »divergent equilibrium« (and/or various taxation schemes, including regressive one) hypothesis in a numerical general equilibrium model of a labor-managed economy[7]. This would also allow one to explore comparative static properties of the system in general equilibrium, an exercise that would be either impossible or uninformative without aid of computer simulation techniques.

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[6] See: Horvat (1986), Domar (1966) and Robinson (1969).

[7] For this line of research, see: Labus (1988) and Bogetić (1990).

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