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# THE COMPLETE OR TOTAL COEFFICIENT OF AGGREGATE DEMAND ELASTICITY

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## INTRODUCTION

The significance of the concept of elasticity and in this context the coefficient of elasticity in economic analysis need not be emphasized particularly. Also, the coefficients of partial elasticity in the case of a function with more variables are an important instrument of quantitative economic analysis. In this paper, however, we will attempt to present the concept of complete or total coefficient of aggregate demand elasticity for the product defined by type and gender. This implies that we will take into consideration the simultaneous change only of those exogenous variables that represent the prices of competitive goods (substitutes) and observe their impact on aggregate demand. The absence of a wide spectrum of other exogenous variables which stipulate the change of aggregate demand only shows that the concept of complete or total coefficient of aggregate demand elasticity in this paper is to be considered conditionally.

## 1. THE CONNECTION BETWEEN THE PARAMETRIC FORM OF THE EQUATION ON THE LINE IN n-DIMENSIONAL REAL EUCLIDIAN SPACE $E^n$ AND THE AVERAGE BASE INDEXES

Assuming the given set of points in $E^n$,

$$
\begin{aligned}
P_o &= (x_{10}, x_{20}, \ldots, x_{no})\\
P_1 &= (x_{11}, x_{21}, \ldots, x_{n1})\\
&\;\;\vdots\\
P_m &= (x_{1m}, x_{2m}, \ldots, x_{nm})
\end{aligned}
$$

and the given scalar equations in parametric form that define the equation of the line i. e. pencil of straight lines trouhl point $P_o = (x_{10}, x_{20}, \ldots, x_{no})$, in $E^n$ as follows,

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$$\left.\begin{aligned} x_{1l}^{(t)} &= x_{10} + \rho^{(t)} \cos \alpha_{1t} \\ x_{2l}^{(t)} &= x_{20} + \rho^{(t)} \cos \alpha_{2t} \\ x_{nl}^{(t)} &= x_{no} + \rho^{(t)} \cos \alpha_{nt} \end{aligned}\right\} \Leftrightarrow x_{il}^{(t)} = x_{io} + \rho^{(t)} \cos\alpha_{it} \begin{pmatrix} i = 1, 2, \ldots, n \\ t = 1, 2, \ldots, m \end{pmatrix} \tag{1}$$

where t denotes arbitrary line of the observed pencil, $x_i^{(t)}{}_l$ the current coordinate of the point of that line, while

$$\cos \alpha_{1t}, \cos \alpha_{2t}, \ldots, \cos \alpha_{nt}, \quad \sum_{i=1}^{n} \cos^2 \alpha_{it} = 1, \ (t = 1, 2, \ldots, m)$$

are the components of the unit vector $\vec{e}^{(t)}$ that is collinear with its corresponding line, and the magnitude $\rho^{(t)}$ given with the formula,

$$\rho^{(t)} = \sqrt{\sum_{i=1}^{n} (x_{il}^{(t)} - x_{io})^2}, \qquad (t = 1, 2, \ldots, m) \tag{2}$$

Assuming that points $P_1, P_2, \ldots, P_m$ lie on the lines $t = 1, 2, \ldots, m$, respectively, then the relevance of the relation

$$\cos \alpha_{it} = \frac{x_{it} - x_{io}}{\Delta\rho^{(t)}}, \quad \Delta\rho^{(t)} = \sqrt{\sum_{i=1}^{n} (x_{it} - x_{io})^2}, \quad (t = 1, 2, \ldots, m) \tag{3}$$

based on which, for $t = 1$ the following equations are also valid

$$\left.\begin{aligned} x_{11} &= x_{10} + \Delta\rho^{(1)} \cos \alpha_{11} \\ x_{21} &= x_{20} + \Delta\rho^{(1)} \cos \alpha_{21} \\ &\ \vdots \\ x_{n1} &= x_{no} + \Delta\rho^{(1)} \cos \alpha_{n1} \end{aligned}\right\} \tag{4}$$

Defining the following averages:

$$\begin{aligned} \bar{x}_1 &= \frac{x_{10} + x_{11} + \ldots + x_{1m}}{m + 1}, \\ \bar{x}_2 &= \frac{x_{20} + x_{21} + \ldots + x_{2m}}{m + 1}, \\ \bar{x}_n &= \frac{x_{no} + x_{n1} + \ldots + x_{nm}}{m + 1}. \end{aligned} \tag{5}$$

Multiplying the system (4) with reciprocal values $(\overline{x}_1 > 0, \ldots, \overline{x}_n > 0)$ from (5) so that we get,

$$\left.\begin{aligned} \frac{x_{11}}{\overline{x}_1} &= \frac{x_{10}}{\overline{x}_1} + \Delta\rho^{(1)} \frac{\cos \alpha_{11}}{\overline{x}_1} \\ \frac{x_{21}}{\overline{x}_2} &= \frac{x_{20}}{\overline{x}_2} + \Delta\rho^{(1)} \frac{\cos \alpha_{21}}{\overline{x}_2} \\ &\;\;\vdots \\ \frac{x_{n1}}{\overline{x}_n} &= \frac{x_{no}}{\overline{x}_n} + \Delta\rho^{(1)} \frac{\cos \alpha_{n1}}{\overline{x}_n} \end{aligned}\right\} \tag{6}$$

Arranging the scalar equations (6) in the following manner,

$$\left(\sum_{i=1}^{n} \frac{x_{i1}}{\overline{x}_i}\right) = \left(\sum_{i=1}^{n} \frac{x_{i0}}{\overline{x}_i}\right) + \Delta\rho^{(1)} \left(\sum_{i=1}^{n} \frac{\cos \alpha_{i1}}{\overline{x}_i}\right) \Big/ \left(\sum_{i=1}^{n} \frac{x_{i0}}{\overline{x}_i}\right)$$

$$\frac{\left[\sum_{i=1}^{n} \frac{x_{i1}}{\overline{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\overline{x}_i}\right]} = 1 + \Delta\rho^{(1)} \frac{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{i1}}{\overline{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\overline{x}_i}\right]} \tag{7}$$

$$\frac{\left[\sum_{i=1}^{n} \frac{x_{i1}}{\overline{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\overline{x}_i}\right]} - 1 = \Delta\rho^{(1)} \frac{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{i1}}{\overline{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\overline{x}_i}\right]} \tag{8}$$

and with analogous repetition of the procedure for $t = 2, \ldots, m$ we would obtain,

$$\frac{\left[\sum_{i=1}^{n} \frac{x_{i2}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} = 1 + \Delta\rho^{(2)} \frac{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{i2}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} \tag{9}$$

$$\frac{\left[\sum_{i=1}^{n} \frac{x_{i2}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} - 1 = \Delta\rho^{(2)} \frac{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{i2}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} \tag{10}$$

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$$\frac{\left[\sum_{i=1}^{n} \frac{x_{im}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} = 1 + \Delta\rho^{(m)} \frac{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{im}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} \tag{11}$$

$$\frac{\left[\sum_{i=1}^{n} \frac{x_{im}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} - 1 = \Delta\rho^{(m)} \frac{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{im}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} \tag{12}$$

or in general

$$\frac{\left[\sum_{i=1}^{n} \frac{x_{it}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} = 1 + \Delta\rho^{(t)} \frac{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{it}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]}, \quad (t = 1, 2, \ldots, m) \tag{13}$$

$$\frac{\left[\sum\limits_{i=1}^{n} \frac{x_{it}}{\bar{x}_i}\right]}{\left[\sum\limits_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]} - 1 = \Delta p^{(t)} \frac{\left[\sum\limits_{i=1}^{n} \frac{\cos \alpha_{it}}{\bar{x}_i}\right]}{\left[\sum\limits_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]}, \quad (t = 1, 2, \ldots, m). \tag{14}$$

Assuming now that $x_{it}$ is the price of product i from the set of homogeneous products in the observed period t, and $x_{io}$ is the price of product i from the set of homogeneous products in the base period. Then $\bar{x}_i$, averages (5), the average price of i-th product for all periods, while the left define the sides of equations (7), (9), (11) or in general (13) appear as the average base indexes. This is one type of complex indexes convenient for following the dynamics of the average price level of homogeneous products[1].

They are obtained by dividing the average price level of homogeneous products in the observed period and the average price level of such products of the base period, or more precisely the division of the average index level of the observed period and the average level of the index chosen for the base[2].

We perceive that the left sides of the equations (8), (10), (12) or in general (14) represent the rate of increase, i. e. the relative change from which is obtained the percentage change of the average price level of homogeneous products in relation to their average level from the base period. At the end of this part of the exposition it should be mentioned that, with a somewhat modified version of the previous procedure, a connection can be established between the parametric form of the aquation of the line in $E^n$ and the price index (Laspeyres' and Paasche's price index), i. e. the cost of living index.

We will not derive this here since our primary task is of another nature.

## 2. CONSTRUCTION OF THE COMPLETE OR TOTAL COEFFICIENT OF AGGREGATE DEMAND ELASTICITY

Assuming that the market is presented with the following parameters:

— Geographical, i. e. the spatial dimension of the market
— Time dimension, i. e. the period of observation
— Present on the market are products for the same purpose (among others)

[1] See: Dr Miloš Blažić: Opšta statistika — osnovi i analiza, Savremena administracija, Belgrade, 1977, p. 200.
[2] For the average index see: Dr Miloš Blažić, op. cit., pp. 198—199.

— By origin these products derive from several (n) mutually independent economic subjects (i. e. do not form an organizational whole).
— Economic subjects have an identical goal, that of achieving commercial benefit from the same payment limited available demand, by independent choice of the type of activity (policies, strategies) on the market. This independence does not imply unallowed activities[3], so that we will state that the market is, under the elements mentioned, manifested through relatively-free economic competition, i. e. as a relatively free market.

Based on the aforegoing, with the expression

$$q = \sum_{i=1}^{n} q_i = f(x_1, x_2, \dots, x_n) \tag{15}$$

the aggregate demand of the set of homogeneous products is defined as the function of prices $x_1, x_2, \dots, x_n$ of these competitive goods, substitutes, i. e. products that satisfy the same need. We are interested how in terms of elasticity demand (15) responds to simultaneous changes of the prices $x_1, x_2, \dots, x_n$.

Concretely we start from the requirement that the relative change of demand (15) becomes a reflection of the relative change of the magnitude that synthesizes in itself the simultaneous changes of prices $x_1, x_2, \dots, x_n$ of homogeneous products. That synthetic magnitude is given in the form of average base indexes, i. e. the rate of increase (14). The complete or total coefficient of aggregate demand elasticity (15) is defined with the expression

$$E_{q}, (x_1, x_2, \dots, x_n) = \frac{\dfrac{\Delta q}{q}}{\dfrac{\left[\sum\limits_{i=1}^{n} \dfrac{x_{it}}{\bar{x}_i}\right]}{\left[\sum\limits_{i=1}^{n} \dfrac{x_{i0}}{\bar{x}_i}\right]} - 1}, \quad (t = 1, 2, \dots, m) \tag{16}$$

or else, at a point

[3] With this we mean the unallowed manner of performing human activites (monopolistic agreements, unfair competition, etc.) in economic competition in the trade of goods and services.

$$E_{q,(x_1,x_2,\dots,x_n)} = \lim_{\substack{(x_{1t}-x_{10})\to 0\\(x_{2t}-x_{20})\to 0\\ \vdots \\ (x_{nt}-x_{n0})\to 0}} \frac{\dfrac{\Delta q}{q}}{\dfrac{\left[\sum_{i=1}^{n}\dfrac{x_{it}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n}\dfrac{x_{i0}}{\bar{x}_i}\right]}-1}, \quad (t = 1,2,\dots,m)^4 \tag{17}$$

Taking into consideration that

$$\frac{\left[\sum_{i=1}^{n}\dfrac{x_{it}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n}\dfrac{x_{i0}}{\bar{x}_i}\right]}-1 = \Delta\rho^{(t)}\frac{\left[\sum_{i=1}^{n}\dfrac{\cos\alpha_{it}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n}\dfrac{x_{i0}}{\bar{x}_i}\right]}, \qquad (t = 1,2,\dots,m)$$

which is really the relation (14), and that the marginal process can be demonstrated through such logical functions as

$$\{\,[\,(x_{1t}-x_{10}\to 0 \wedge (x_{2t}-x_{20})\to 0 \dots \wedge (x_{nt}-x_{n0})\to 0] \Leftrightarrow \Delta\rho^{(t)}\to 0\,\}\,, \qquad (t = 1,2,\dots,m) \tag{18}$$

then the expression (17) becomes

$$E_{q,(x_1,x_2,\dots,x_n)} = \lim_{\Delta\rho^{(t)}\to 0} \frac{\dfrac{\Delta q}{q}}{\dfrac{\left[\sum_{i=1}^{n}\dfrac{\cos\alpha_{it}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n}\dfrac{x_{io}}{\bar{x}_i}\right]}\cdot\Delta\rho^{(t)}}, \quad (t = 1,2,\dots,m) \tag{19}$$

[4] In this marginal process it is taken that $(x_{1t}-x_{10},\ x_{2t}-x_{20},\dots,\ x_{nt}-x_{n0}) \to (0,\ 0,\ \dots,\ 0)$ along the line t.

$$= \frac{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{it}}{\bar{x}_i}\right]} \frac{1}{q} \lim_{\Delta\rho^{(t)} \to 0} \frac{\Delta q}{\Delta \rho^{(t)}}, \qquad (t = 1, 2, \ldots, m) \qquad (20)$$

from which for the differentiable function q we obtain

$$E_{q, (x_1, x_2, \ldots, x_n)} = \frac{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{it}}{\bar{x}_i}\right]} \frac{1}{q} \times$$

$$\left[\sum_{i=1}^{n} \frac{\partial q}{\partial x_i} \cos \alpha_{it}\right]_{p_o}^{5)}, \qquad (t = 1, 2, \ldots, m) \qquad (21)$$

Since $\cos \alpha_{it}$ can be calculated as

$$\cos \alpha_{it} = \frac{x_{it} - x_{i0}}{\Delta \rho^{(t)}}, \qquad \begin{matrix} i = 1, 2, \ldots, n \\ t = 1, 2, \ldots, m \end{matrix}$$

we can further tranform (21) to

$$E_{q, (x_1, x_2, \ldots, x_n)} = \frac{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{it}}{\bar{x}_i}\right]} \frac{1}{\Delta \rho^{(t)}} \frac{1}{q} \times$$

$$\left[\sum_{i=1}^{n} \frac{\partial q}{\partial x_i} (x_{it} - x_{i0})\right]_{p_o}, \qquad (t = 1, 2, \ldots, m) \qquad (22)$$

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[5] Point $(x_{1t}, x_{2t}, \ldots, x_{nt})$, $t = 1, 2, \ldots, m$ moves towards point $(x_{10}, x_{20}, \ldots, x_{n0})$ along the arbitrary line, t, and thus

$$\lim_{\Delta p^{(t)} \to 0} \frac{\Delta q}{\Delta p^{(t)}} = \left[ \sum_{i=1} \frac{\partial q}{\partial x_i} \cos\alpha_{it} \right]_{P_o}$$

where the partial derivatives are taken at point $P_o = (x_{10}, x_{20}, \ldots, x_{no})$. or else,

$$E_{q, (x_1, x_2, \ldots, x_n)} = \frac{1}{\Delta \rho^{(t)} \cdot \dfrac{\left[ \sum_{i=1}^{n} \dfrac{\cos \alpha_{it}}{\bar{x}_i} \right]}{\left[ \sum_{i=1}^{n} \dfrac{x_{i0}}{\bar{x}_i} \right]}} \frac{d^{(t)}q \mid P_o}{q} \qquad (t = 1, 2, \ldots, m) \qquad (23)$$

In formula (23) the expression $d^{(t)}q| P_o$ represents the complete or total first-order differential at point $P_o = (x_{10}, x_{20}, \ldots, x_{n0})$ for increases,

$$(x_{1t} - x_{10}), \; (x_{2t} - x_{20}), \ldots, \; (x_{nt} - x_{n0}), \qquad (t = 1, 2, \ldots, m).$$

Finally, we can write that

$$E_{q, (x_1, x_2, \ldots, x_n)} = \frac{1}{\dfrac{\left[ \sum_{i=1}^{n} \dfrac{x_{it}}{\bar{x}_i} \right]}{\left[ \sum_{i=1}^{n} \dfrac{x_{i0}}{\bar{x}_i} \right]} - 1} \frac{d^{(t)}q \mid P_o}{q}, \qquad (t = 1, 2, \ldots, m) \qquad (24)$$

or shorter,

$$E_{q, (x_1, x_2, \ldots, x_n)} = \frac{1}{(I_t - 1)} \frac{d^{(t)}q \mid P_o}{q}, \qquad (t = 1, 2, \ldots, m) \qquad (25)$$

where $I_t$ will denote the average base indexes. One can easily demonstrate that (25) has another equivalent form of appearance.

Namely,

$$E_{q, (x_1, x_2, \ldots, x_n)} = \frac{1}{(I_t - 1)} \left[ \sum_{i=1}^{n} \frac{x_{i0}}{q} \frac{\partial q}{\partial x_i} \frac{(x_{it} - x_{i0})}{x_{i0}} \right]_{P_o},$$

$$(t = 1, 2, \ldots, m) \qquad (26)$$

$$= \frac{1}{(I_t - 1)} \left[ \sum_{i=1}^{n} \left( \frac{x_{it} - x_{i0}}{x_{i0}} \right) E_{q, x_i} \right]_{P_o}, \qquad (t = 1, 2, \ldots, m) \tag{27}$$

$$E_{q, (x_1, x_2, \ldots, x_n)} = \left[ \sum_{i=1}^{n} \frac{\left( \frac{x_{it} - x_{i0}}{x_{i0}} \right)}{(I_t - 1)} E_{q, x_i} \right]_{P_o},$$

$$(t = 1, 2, \ldots, m) \tag{28}$$

The coefficients of partial elasticity contained in formula (28), $E_{q,xi}$ are taken at point $P_o$, and weighted by the ratio of the rate of increase of the price of the concrete product and rate of increase (14). The complete or total coefficient of elasticity (16), i. e. (25) and (28) show by how many percentage points demand (15) changes in relation to the previous level for the one percent change of the average price level of homogeneous products in relation to their average level from the base period.

Of course, the relative change in demand (15) can also be observed in simultaneous, unsynthetic relative changes of the prices $x_1$, $x_2, \ldots, x_n$, but one then commences from the expression

$$\frac{d^{(t)} q}{q} = \left[ \sum_{i=1}^{n} \frac{x_{i0}}{q} \frac{\partial q}{\partial x_i} \frac{x_{it} - x_{i0}}{x_{i0}} \right]_{P_o,} \qquad (t = 1, 2, \ldots, m) \tag{29}$$

$$= \left[ \sum_{i=1}^{n} \left( \frac{x_{it} - x_{i0}}{x_{i0}} \right) E_{q, x_i} \right]_{P_{o,}} \qquad (t = 1, 2, \ldots, m) \tag{30}$$

that represents the infinitesimal relative change of coordinate q (demand of the tangential hyper plane on the hyper surbface (15) at point $P_o$, when, starting from point $P_o$,

$x_1$ changes for $\frac{x_{1t} - x_{10}}{x_{10}}$ 100%, $\qquad t = 1, 2, \ldots, m$

$x_2$ changes for $\frac{x_{2t} - x_{20}}{x_{20}}$ 100%, $\qquad t = 1, 2, \ldots, m$

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$x_n$ changes for $\frac{x_{nt} - x_{n0}}{x_{n0}}$ 100%, $\qquad t = 1, 2, \ldots, m$

## 3. ANALYSIS OF THE COMPLETE OR TOTAL COEFFICIENT OF AGGREGATE DEMAND ELASTICITY

Assuming t = 1 and that the following conditions are valid,

$$\left.\begin{aligned} & x_{11} \neq x_{10} \\ & x_{21} = x_{20} \Rightarrow \frac{x_{21}}{\bar{x}_2} = \frac{x_{20}}{\bar{x}_2} = a_2 > 0 \\ & x_{31} = x_{30} \Rightarrow \frac{x_{31}}{\bar{x}_3} = \frac{x_{30}}{\bar{x}_3} = a_3 > 0 \\ & \cdot \\ & \cdot \\ & \cdot \\ & x_{n1} = x_{n0} \Rightarrow \frac{x_{n1}}{\bar{x}_n} = \frac{x_{n0}}{\bar{x}_n} = a_n > 0 \end{aligned}\right\} \tag{31}$$

i. e, that the prices of product i = 2, 3, . . . , n in period t = 1 remain unchanged in relationship to the base period, apart from the price of product i = 1. In this case, we will attempt to observe and analyse the constituent elements of coefficient (25), i. e. (28). Expression (28), due to the conditions (31) is

$$E_{q,(x_1, x_2, \dots, x_n)} = \frac{\dfrac{x_{11} - x_{10}}{x_{10}}}{I_1 - 1} E_{q, x_1} \tag{32}$$

while

$$I_1 - 1 = \frac{\dfrac{x_{11}}{\bar{x}_1} + \dfrac{x_{21}}{\bar{x}_2} + \dots + \dfrac{x_{n1}}{\bar{x}_n}}{\dfrac{x_{10}}{\bar{x}_1} + \dfrac{x_{20}}{\bar{x}_2} + \dots + \dfrac{x_{n0}}{\bar{x}_n}} - 1,$$

[6] In measurement technique, i. e. approximate calculus the ratio of the complete or total first-order differential to the function of which is formed is called a relative error. On relative errors see: T. Pejović, Matematička analiza I, Naučna knjiga, Belgrade, 1960, p. 255.

$$= \frac{\dfrac{x_{11}}{\bar{x}_1} + a_2 + \ldots + a_n}{\dfrac{x_{10}}{\bar{x}_1} + a_2 + \ldots + a_n} - 1.$$

$$= \frac{x_{11} + \bar{x}_1 (a_2 + a_3 + \ldots + a_n)}{x_{10} + \bar{x}_1 (a_2 + a_3 + \ldots + a_n)} - 1,$$

$$= \frac{x_{11} + b}{x_{10} + b} - 1, \qquad b = x_1 (a_2 + a_3 \ldots + a_n) > 0$$

$$= \frac{x_{11} - x_{10}}{x_{10} + b} \tag{33}$$

Interpolating (33) in (32) gives

$$E_{q, (x_1, x_2, \ldots, x_n)} = \frac{\dfrac{x_{11} - x_{10}}{x_{10}}}{\dfrac{x_{11} - x_{10}}{x_{10} + b}} E_{q, x_1}$$

$$= \frac{x_{10} + b}{x_{10}} E_{q, x_1}$$

$$= \left( 1 + \frac{\bar{x}_1 (a_2 + a_3 + \ldots + a_n)}{x_{10}} \right) E_{q, x_1} \tag{34}$$

and when the values for $a_2, a_3, \ldots, a_n$ from (31) are interpolated in (34), after arranging, the expression (34) becomes

$$E_{q, (x_1, x_2, \ldots, x_n)} = \left( 1 + \frac{\bar{x}_1}{\bar{x}_2} \frac{x_{20}}{x_{10}} + \frac{\bar{x}_1}{\bar{x}_3} \frac{x_{30}}{x_{10}} + \ldots + \frac{\bar{x}_1}{\bar{x}_n} \frac{x_{n0}}{x_{10}} \right) E_{q, x_1} \tag{35}$$

or else,

$$E_{q,(x_1,x_2,\ldots,x_n)} = \left(1 + \frac{\dfrac{x_{20}}{x_{10}}}{\dfrac{\bar{x}_2}{\bar{x}_1}} + \frac{\dfrac{x_{30}}{x_{10}}}{\dfrac{\bar{x}_3}{\bar{x}_1}} + \ldots + \frac{\dfrac{x_{n0}}{x_{10}}}{\dfrac{\bar{x}_n}{\bar{x}_1}}\right) Eq,\ x_1 \tag{36}$$

Elements

$$\frac{x_{20}}{x_{10}}, \frac{x_{30}}{x_{10}}, \ldots, \frac{x_{n0}}{x_{10}} \tag{37}$$

used in expression (36) represent the price relatives of the base period while ratios

$$\frac{\bar{x}_2}{\bar{x}_1}, \frac{\bar{x}_3}{\bar{x}_1}, \ldots, \frac{\bar{x}_n}{\bar{x}_1} \tag{38}$$

constitute price relatives on the basis of average values. The ratios of the price relatives (37) and (38), i. e.

$$\frac{\dfrac{x_{20}}{x_{10}}}{\dfrac{\bar{x}_2}{\bar{x}_1}}, \frac{\dfrac{x_{30}}{x_{10}}}{\dfrac{\bar{x}_3}{\bar{x}_1}}, \ldots, \frac{\dfrac{x_{n0}}{x_{10}}}{\dfrac{\bar{x}_n}{\bar{x}_1}} \tag{39}$$

revael the level or position of price relatives of the base period in relation to price relatives on an average basis, and also simultaneously weight the partial coefficient of elasticity $Eq, x_1$. Hence, if the direction of the structural change of prices $x_1, x_2, \ldots, x_n$ is given with relation (31) then the one-percent change of the average level of these prices in relation to their average level from the base period determines the relative change in demand (15) in the amount given with expression (36). We can analyse questions of range, i. e. the maximum and minimum relative change in demand (15) through formula (21) which is the most convenient for this task. Accordingly,

$$E_{q,(x_1,x_2,\ldots,x_n)} = \frac{\left[\sum\limits_{i=1}^{n} \dfrac{x_{i0}}{\bar{x}_i}\right]}{\left[\sum\limits_{i=1}^{n} \dfrac{\cos\alpha_{it}}{\bar{x}_i}\right]} \; \frac{1}{q} \left[\sum_{i=1}^{n} \frac{\partial q}{\partial x_i} \cos\alpha_{it}\right]_{P_o}$$

$$= \frac{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{\cos \alpha_{it}}{\bar{x}_i}\right]} \frac{1}{q} [grad\ q \cdot \vec{e}^{(t)}]^{7)} \tag{40}$$

and since in general the speed of change of the value of the function at an arbitrary point in the direction of the gradient is at its maximum, this means that in our case the maximum of the relative change in demand (15) is determined with the kind of direction of change in prices $x_1, x_2, \ldots, x_n$ for which vector $\vec{e}^{(t)}$ has the components

$$\cos \alpha_{it} = \frac{x_{it} - x_{i0}}{\Delta \rho^{(t)}} = \frac{\left(\frac{\partial q}{\partial x_i}\right)_{P_o}}{|grad\ f|}, \qquad \begin{array}{l} i = 1, 2, \ldots, n \\ t = 1, 2, \ldots, m \end{array} \tag{41}$$

so that expression (40) becomes

$$E_{q, (x_1, x_2, \ldots, x_n)} = \frac{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]}{\frac{1}{|grad\ f|}\left[\sum_{i=1}^{n} \frac{\left(\frac{\partial q}{\partial x_i}\right)_{P_o}}{\bar{x}_i}\right]} \frac{1}{q} |grad\ f| \tag{42}$$

---

7 $$\left[\sum_{i=1}^{n} \frac{\partial q}{\partial x_i} \cos \alpha_{it}\right]_{P_o} = grad\ q \cdot \vec{e}^{(t)} = |grad\ q| \cdot \cos \sphericalangle (grad\ q, \vec{e}^{(t)}) =$$

$proj_{\vec{e}^{(t)}}\ grad\ q$ since $\vec{e}^{(t)}$ is of unit intensity.

$$E_{q,(x_1,x_2,\dots,x_n)} = \frac{\left[\sum_{i=1}^{n} \frac{x_{i0}}{\bar{x}_i}\right]\left[\sum_{i=1}^{n}\left(\frac{\partial q}{\partial x_i}\right)^2_{P_o}\right]}{\left[\sum_{i=1}^{n} \frac{\left(\frac{\partial q}{\partial x_i}\right)_{P_o}}{x_i}\right]} \frac{1}{q} \tag{43}$$

If, on the other hand, the direction of change of prices $x_1, x_2, \dots, \dots, x_n$ of competitive goods is such that it causes,

$$grad\ q \cdot \overrightarrow{e^{(t)}} = proj_{\overrightarrow{e^{(t)}}}\ grad = 0 \Longrightarrow E_{q,(x_1,x_2,\dots,x_n)} = 0 \tag{44}$$

then in interpreting this case one cannot resort only to the classical economic interpretation of perfectly inelastic demand. That is, the one-percent change of the average price level of competitive goods in relation to their average level from the base period, synthesizes the change in prices $x_1, x_2, \dots, x_n$ with consequence (44), i. e. the relative demand change is equal zero, and the absolute level of aggregate demand (15) remains unchanged. In effect, we have the case of demand redistribution (15), i. e. the change of market share of the competitors in the form of an unchanged level of aggregate demand (15).

## 4. CONCLUDING REMARKS

As we have seen, the key elements in constructing the complete or total coefficient of aggregate demand elasticity represented the average base indexes, i. e. the rate of increase (14), the magnitude that synthesizes the simultaneous change of prices $x_1$, $x_2, \dots, x_n$. We also established (upon moving to the marginal process) that the relative change of aggregate demand at a one-percent change of the average price level of homogeneous products in relation to their average level from the base period, is the function of the direction of change of these prices. If, for the sake of simplicity, we observe the prices of three competitive products $x_1, x_2, x_3$ given by points $P_o$, $P_1$, $P_2$, $P_3$, $P_4$ then the average base indexes represent the directions of change of these prices, as shown in fig 1(a).

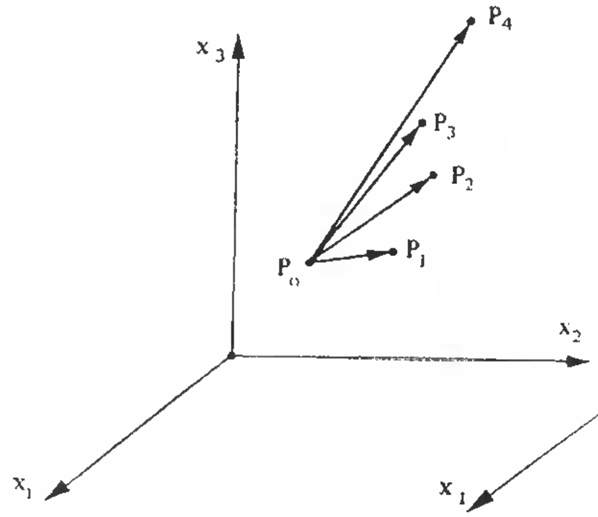


Fig. 1(a)

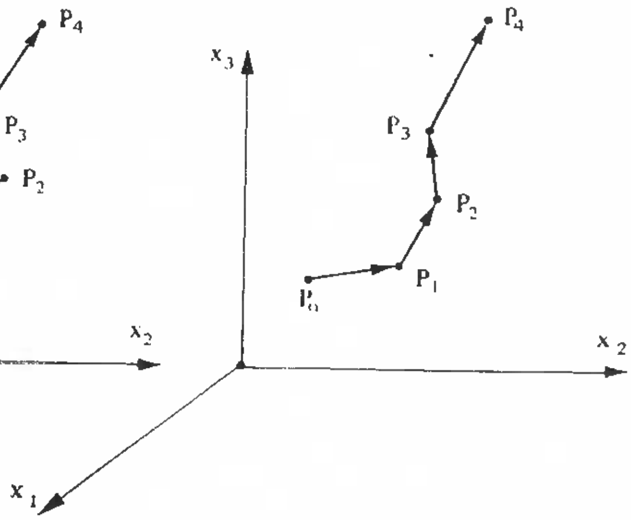


Fig. 1(b)

Point $P_0$ is obviously the starting point, the reference level (base period) in relation to which all changes are observed. The lack of a complete or total coefficient of elasticity is obvious. In other words, it does not tell us anything about the size of relative changes of aggregate demand between two periods, the one observed and the one preceding it, which again based on our simplified example implies that the directions of change of prices shown in Fig. 1(b) are not taken into consideration. This deficiency is only of minor relevance, in other words removable. It is enough to include the average chain indexes.[8]

$$I_{tl} = \frac{\left[\sum_{i=1}^{n} \frac{x_{it}}{\bar{x}_i}\right]}{\left[\sum_{i=1}^{n} \frac{x_{i(t-1)}}{\bar{x}_i}\right]}, \qquad (t = 1, 2, \ldots, m)$$

into consideration, analogous to the already presented procedure for average base indexes, so that the complete or total coefficient of aggregate demand elasticity gains a dimension that it lacked. Finally, we emphasize (even though it is implicitly contained in the presentation) that the construction, i. e. the analysis and interpretation of the complete or total coefficient of aggregate demand elasticity, has been made under the assumption of a constant, unchangeable income level from which demand is financed (15).

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[8] For the average chain indexes see: Dr Miloš Blažić, op. cit., 200—201.

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## *POTPUNI ILI TOTALNI KOEFICIJENT ELASTIČNOSTI AGREGATNE TRAŽNJE*

*Slobodan SEKULOVIĆ*

### *Rezime*

*Značaj pojma elastičnosti te s tim u vezi koeficijenta elastičnosti u ekonomskoj analizi nije potrebno posebno naglašavati. Isto tako, koeficijenti parcijalne elastičnosti u uslovima funkcije više promjenljivih važan su instrumentarij kvantitativne ekonomske analize. U ovom radu, pak, pokušaćemo izložiti koncept potpunog ili totalnog koeficijenta elastičnosti agregatne tražnje za proizvodom koji je određen po vrsti, rodu (generički). To znači da ćemo uzeti u obzir istovremenu promjenu, samo onih egzogenih varijabli koje predstavljaju cijene konkurentskih dobara (supstituta) i posmatrati njihov uticaj na agregatnu tražnju. Izostavljanje širokog spektra ostalih egzogenih varijabli koje uslovljavaju promjenu agregatne tražnje, samo govori da termin potpuni ili totalni koeficijent elastičnosti agregatne tražnje treba u ovom radu shvatiti uslovno.*