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A MODEL WITH MANY CONSUMER GOODS AND ONE CAPITAL GOOD

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A. CHANGING LABOUR

Quantity equations

Let the economy produce two consumer goods (x_1, x_2) and one producer good (x_3). Labour is expanding (contracting) at a constant rate g. Consequently, with unchanged lifespan of machines (n), the replacement rate is also constant, $\delta = \delta(g)$. The quantity equations are as follows:

$$\lambda_1 X_1 + \lambda_2 X_2 + \lambda_3 X_3 = L \tag{1}$$

$$r\kappa_1 X_1 + r\kappa_2 X_2 + r\kappa_3 = rK = X_3 \qquad r = \delta + g$$

With three variables and only two equations, there is one degree of freedom. Set $X_1 = 1$ so that all variables are expressed as ratios

$$\left(\frac{X_2}{X_1}, \frac{X_3}{X_1} \text{ and also } \tilde{L} = \frac{L}{X_1}\right)$$

$$\frac{X_2}{X_1} = \frac{(1-r\kappa_3)\,\tilde{L}}{\lambda_2 + rm_2} - \frac{\lambda_1 + rm_1}{\lambda_2 + rm_2} \qquad m_1 = \kappa_1\lambda_3 - \kappa_3\lambda_1$$

$$m_2 = \kappa_2\lambda_3 - \kappa_3\lambda_2$$

$$\frac{X_3}{X_1} = \frac{r\kappa_2\tilde{L}}{\lambda_2 + rm_2} + \frac{rm_3}{\lambda_2 + rm_2} \qquad m_3 = \kappa_1\lambda_2 - \kappa_2\lambda_1 \tag{2}$$

With three industries, there are three mechanization possibilities (m_1, m_2, m_3) and they all appear in the second fractions. Note that ra-

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tios are technologically determined except for the element $\tilde{L} = L/X_1$. If X_1 increases, given labour force, $\tilde{L}$ decreases and so do X_2/X_1 and X_3/X_1; the composition of demand may change even if the stock of machines expands at the constant gross rate r. Since the proportion X_2/X_1 is a decreasing function of X_1, the composition of supply of consumer goods may be adjusted to accomodate any structure of demand. Changes in demand do *not* require any change in technology. We shall assume, however, that consumer preferences are a function of exclusively per capita income. Without technological progress, per capita income does not change and so the structure of demand remains unchanged. This renders $\tilde{L} = L/X_1$ and all other ratios unchanged and the system is fully determined.

In general, with n consumer goods and one machine, there will be n—1 degrees of freedom in the system and they may be used to match the changing demand. If the structure of demand remains unchanged, all supply ratios X_i/X_j remain unchanged as well and absolute outputs X_i are fully determined by the technology used and the size of the labour force L.

Now, if the structure of consumption does not change, we can always define a composite consumer good B whose price is p_B and so for n consumer goods

$$\sum_{i=1}^{n} p_i X_i = p_B B = L$$

B is of course, the output of our standard baskets in the two-industry case. Thus, n consumer goods under uniform growth do not change the initial model in any substantial way.

Price equations

Unlike the asymmetry in the quantity equations, there are as many price equations as there are prices and so prices are always fully determined.

$$\begin{aligned} r p_3 \kappa_1 + \lambda_1 &= p_1 \\ r p_3 \kappa_2 + \lambda_2 &= p_2 \\ r p_3 \kappa_3 + \lambda_3 &= p_3 \end{aligned} \qquad (3)$$

As there is only one machine industry, the price of machine (p_3) is determined exclusively by the technology of its own industry. The other two prices are now functions of two relative degrees of mechanization.

$$p_1 = \frac{\lambda_1 + rm_1}{1 - r\kappa_3}$$

$$p_2 = \frac{\lambda_2 + rm_2}{1 - r\kappa_3} \qquad m_1 = \kappa_1\lambda_3 - \kappa_3\lambda_1 \tag{4}$$

$$m_2 = \kappa_2\lambda_3 - \kappa_3\lambda_2$$

$$p_3 = \frac{\lambda_3}{1 - r\kappa_3} \tag{5}$$

$$\frac{p_1}{p_2} = \frac{\lambda_1 + rm_1}{\lambda_2 + rm_2}, \qquad \frac{p_i}{p_3} = \frac{\lambda_1 + rm_i}{\lambda_3}, \qquad i = 1,2$$

Relative prices are equal to ratios of direct labour coefficients if either $r = 0$ or the degree of mechanization in the respective industries are the same as in the machine industry. This is a general result and applies to any number of consumer good industries.

A rational consumer will spend his income in such a way as to maximize his utility. In the optimum situation the ratios of marginal utilities of any two goods consumed will be equal to the ratios of their prices. Since prices are fully determined by the technology of the system, the goods will be consumed in the proportions that will equalize the rates of commodity substitution with the price ratios for all consumers.

B. TECNOLOGICAL PROGRESS

Output growth and demand changes

Technological progress increases per capita income and changes prices. Even with constant consumer preferences, the structure of demand will change due to different income and price elasticities. New and luxury goods will expand faster, old goods and necessities will expand at a lower rate, and the demand for inferior goods will shrink. All these changes can be accommodated by the available degrees of freedom.

A neutral technological progress will leave relative prices unchanged. But per capita income will rise and different income elasticities will change the structure of demand. Thus, there is no point in studying particular patterns of technological progress. We shall assume a completely general case with technological progress changing all prices and quantities differently. Technological progress is unembodied.

If the structure of demand changes, each industry will grow at its own rate of growth h_i. As a result, the sectoral rates of replacement will be different as well. The queistion arises: what is the appropriate rental rate? In answering the question, we treat the economy as a unified production system and look for an optimum allocation of resources within that system.

Assume that labour force remains unchanged, L = const., and technological progress affects only labour coefficients: it increases productivity of live labour differently in each sector, which implies that within a certain period each labour coefficient will decrease by a factor $\Gamma_i = (1 + \gamma_i)$. By the end of the period the labour balance in (1) assumes the following form

$$\lambda_1 \Gamma_1^{-1} X_1 + \lambda_2 \Gamma_2^{-1} X_2 + \lambda_3 \Gamma_3^{-1} X_3 = \Gamma^{-1} L \qquad (6)$$

where Γ represents an average increase of labour productivity in the system. Because of productivity increase, $L - \Gamma^{-1} L = L(1 - \Gamma^{-1})$ workers will become redundant. In order to restore full employment, output of each industry must increase by a factor $H_i = 1 + h_i$.

$$\lambda_1 \Gamma_1^{-1} X_1 H_1 + \lambda_2 \Gamma_2^{-1} X_2 H_2 + \lambda_3 \Gamma_3^{-1} X_3 H_3 = (\Gamma^{-1} L)\, \Gamma = L \qquad (7)$$

If capital coefficients do not change, the investment balance in (1) becomes

$$(\delta_1 + h_1)\, \kappa_1 X_1 + (\delta_2 + h_2)\, \kappa_2 X_2 + (\delta_3 + h_3)\, \kappa_3 X_3 = (\delta + h)\, K = X_3 \qquad (8)$$

where hK is the number of new machines necessary to equip

$$L(1 - \Gamma^{-1}) = \frac{\gamma}{1 + \gamma} L$$

redundant workers.
We have three variables H_i, i = 1, 2, 3, and two equations: (7) and (8). We still need one equation. When technological progress increases per capita net output, the structure of demand becomes a function of income (L^* with higher productivity, $L^* = p_1 H_1 X_1 + p_2 H_2 X_2$), prices ($p_1, p_2$) and consumer preferences which are assumed to be given and unchangeable. Prices are fully determined by technology (T). Thus, the proportion in which the two goods are consumed is fully determined by income and technology.

As X_1 and X_2 are initial outputs before technological change, they are known, and so we may write

$$\frac{H_2}{H_1} = \sigma (L^*, T) \qquad (9)$$

The proportion σ adds the missing equation; the system is closed and all outputs are fully determined. If there are *n* consumer goods, there will be n — 1 proportions $\sigma_i = H_i / H_1$, i = 2, ... n, which will provide an equal number of missing equations. Adding labour and investment balances (7) and (8), we obtain n + 1 equations which determine the output of *n* consumption goods and one machine.

The Pasinetti system

At the beginning of the period, labour and investment (for the period) equations are as follows:

Labour equation: $\lambda_1 X_1 + \lambda_2 X_2 + \lambda_3 X_3 = L$ (10)

Investment equation: $r_1 \kappa_1 X_1 + r_2 \kappa_2 X_2 + r_3 \kappa_3 X_3 = rK = X_3, \; r_i = \delta_i + h_i$

Summing up the components of two equations evaluated in corresponding prices, we obtain value balances. Dividing by X_1, we obtain price equations

$r_i p_3 k_i + w\lambda_i = p_i, \qquad i = 1, 2, 3$ (11)

The system (11) will be called a Pasinetti system.* Pasinetti himself calls it »the natural economic system«, but there is nothing particularly natural in it. Pasinetti justifies the approach by pointing out that in this case costs of production represent vertically hyperintegrated labour inputs. That is true, but irrelevant. It only means that each industry finances its own ivnestment costs. But why should it? The resulting different rates of profit lead to a misallocation of resources. A higher than average rate of profit means a higher price. By reallocating a unit of labour from less profitable to more profitable industry the welfare of the community is increased because the ratios of prices determine the ratios of marginal utilities. An optimum is achieved when profits are equalized throughout the economy.

The equalization of the rates of profit

Equal profit rates imply equal depreciation charges, which are simply functions of n and h. That may look puzzling, because physical replacement differs from industry to industry. The explanation is simple: an economy must produce *physical* machines to replace an equal number of scrapped machines in the corresponding period; an individual firm faces no physical constraint of this sort. A firm must produce *value* to cover depreciation charges. Whether a firm expands fast or slow, the depreciation rate must be such that depreciation charges accumulated at a common interest rate h will generate value equal to the purchase price of a new machine which will replace the old one at the time it is scrapped. Clearly, if the stock of machines is growing at the rate h, the sum of all replacements and the sum of all depreciations will be equal because they are the same functions of the same parameters n and h.

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* Compared with the actual model worked out by Pasinetti, there are two technical differences. Instead of replacement rates $\delta_i = 1/\nu_i$, Pasinetti uses linear depreciation rates $1/n_i$ ($1/T_i$ in his notation). And machines are produced only by labour and not by other machines. The former is erroneous and the second is an unnecessary simplification. (Pasinetti, 1981, chps. VII and VIII).

We have just found that maximization of economic welfare requires an equalization of profit rates. Does that conflict with the formation of labour values? The answer is: no. Consistently derived labour prices also require an equalization of profit rates. Profits are nothing else but investment costs which must be incurred in order to equip labour rendered redundant by technological progress. If g is a uniform cost incurred by the growth of live labour, h represents the growth of embodied labour when live labour remains constant under technological progress. Both g and h represent systemic costs and, therefore, must be uniform throughout the system.

The uniform, i. e. average, profit and depreciation rates are easily formed:

$$h = \frac{\Sigma h_i K_i}{K}, \qquad \delta = \frac{\Sigma \delta_i K_i}{K}$$

If the depreciation charges $\delta_t K$ are accumulated over n years at the interest rates h, they will provide an investment fund exactly equal to K. This can be proved in the following way.

At the beginning of the period $t = n + 1$ the number of machines in the capital stock will be equal to past n investments. Investments increase by unequal factors H_t per period. Starting with a unit investment at $t = 1$, the number of machines will increase by the end of $t = n$ to

$$Kn = 1 + H_1 + H_1 H_2 + \dots + \prod_{\tau=1}^{n-1} H_\tau$$

In $t = n + 1$ the first investment will be replaced. $R_{n+1} = 1$. Consequently, the replacement rate is equal to

$$\delta_{n+1} = \frac{R_{n+1}}{K_n} = \frac{1}{1 + H_1 + \dots + \prod_{\tau=1}^{n-1} H_\tau}$$

which may also be written in the following way

$$R_{n+1} (1 + H_1 + H_1 H_2 + \dots + \prod_{\tau=1}^{n-1} H_\tau) = K_n$$

Depreciation calculated at $t = 1$,

$$D = \frac{R_{n+1}}{\prod_{\tau=1}^{n-1} H_\tau}$$

accumulated in n installements at successive rates h_τ must amount to K_n

$$D \left(\prod_{\tau=1}^{n-1} H_\tau + \prod_{\tau=2}^{n-1} H_\tau + \ldots + H_{n-1} + 1 \right) = K_n \qquad Q.E.D.$$

All that remains is to write down the price equations:

$$\begin{aligned} p_3(\delta+h)\,\kappa_1 + w\lambda_1 &= p_1 \\ p_3(\delta+h)\,\kappa_2 + w\lambda_2 &= p_2 \\ p_3(\delta+h)\,\kappa_3 + w\lambda_3 &= p_3 \end{aligned} \qquad (12)$$

w = 1 implies labour prices

Four important consequences follow:

1. The economy may be decentralized because the profit maximizing firms will move into high-profit industries until profits are equalized throughout the economy.

2. Gross investment in (8) will not change if we put

$$\delta_i + h_i = \delta + h = r$$

But sectoral investment outlays (δ_i+h_i) and sectoral investment finance $(\delta+h)$ are no longer equal which necessitates the establishment of a financial sector in the economy. Money is introduced in the system in a natural way. It becomes a consequence of the optimizing behaviour.

3. In the process, prices will change: in the more profitable industries prices will decrease, in the less profitable they will increase.

4. Since prices determine the choice of technology, different techniques will be chosen (substitution activities) in order to minimize costs. As a result, all the parameters of the system will change: labour (and capital) coefficients because of a different technology and replacement and growth rates because of different sectoral growth. Thus, technological progress generates two different types of changes: (1) improvements in technology and (2) substitution of techniques.

If also capital coefficients are changed, h (and, consequently, δ) will change as well. If capital coefficients are reduced, less capital will be necessary and h will be reduced. A reduction of capital coefficients may compensate (as in Neutral II TP) or even overcompensate the investment expanding effects of reduced labour coefficients, rendering h zero or negative.

The growth of labour force simultaneously with technological progress represents no difficlty either. All labour costs increase by investment outlays necessary to equip new workers, i. e., by gk_i. . Thus, fully generally, the price equations look as follows

$$p_{n+1} (\delta + g + h) k_i + \lambda_i = p_i, \qquad i = 1, \ldots n + 1$$

where p_{n+1} is the price of the single capital good.

Since capital goods are evaluated in labour prices, $\pi = g + h$ indicates additional labour embodied in new machines.

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