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FINANCING OF PUBLIC SERVICES IN YUGOSLAVIA: A LINDAHL EQUILIBRIUM MODEL FOR THE LABOUR-MANAGED ECONOMY[1]

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I INTRODUCTION

The Yugoslav economic system is characterized by the dominance of social property over productive capital and by a combination of market and plan as co-ordinating mechanisms for resource allocation, the latter serving a primarily indicative role. Firms producing and distributing public goods are, from the formal legal point of view, no different than the firms producing and distributing consumer goods.[1]

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[1] Some definitions used in this paper:

a) Something is a *good* if, ceteris paribus, any individual prefers to consume more of it rather than less, or if it is used in production of such a commodity. (This definition is due to Foley [3], p. 48.)

b) *Exchange goods* are goods and services which, when consumed by one person in society, cannot be consumed by any other person in society. These are all consumer goods and services, and all inputs in production that are bought and sold in ordinary markets. The synonym that will be used in this paper is the *consumer good*, while the synonym most often used in the literature is the *private good*.

c) By *non-exchange goods* or *public goods* we mean those goods for which the amounts consumed by the members of society are all equal to each other and to the total produced. This definition is due to Samuelson [13], p. 387. He pointed out that this fact was dictated by technology (e. g., national defense), rather than by policy (e. g., public schooling). See [14].

Note that in the case of exchange goods, the idea of consumption is directly related to the idea of »destruction« of something by individual use. For this reason, the amount consumed by one person is not available for consumption by another person. In case of non-exchange goods, everyone consumes the same total amount of the good produced, so that an increase in one person's consumption, or adding an additional consumer, while keeping the aggregate expenditure on the good constant, does not decrease the consumption of other people by the same amount.

d) *Pure public good* refers to non-market output without a welfare content, which represents a social overhead cost (judiciary, police, army, public administration). These are the non-exchange goods for which it is

They are run by all the firms's employees on the one-man one-vote basis, the capital they use in producing the output is socially owned,[2] and the net income earned by selling their products is freely distri-

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not possible to exclude any person from consumption, and therefore it is not possible to require consumers to pay for the good.

e) *Collective good* refers to non-market output with welfare content (education, medical care, social welfare, culture, physical culture, environmental creation and conservation). These are the non-exchange goods for which the exclusion from consumption is possible, and which could, therefore, be supplied through a private market. In this sense, collective goods cover intermediate cases between the two extremes, the consumer goods and the pure public goods, i. e., the cases such that, given the existing supply of a collective good (e. g., a highway between places Z and S), each additional consumer reduces the other consumers' consumption by only a little. See Atkinson and Stiglitz [1], pp. 484—485.

f) *Local public goods* are the non-exchange goods whose benefits are confined to a particular geographic location or community, within which they are available at no additional cost to new residents (e. g., local infrastructure — water and power supplies, local roads, public transport, etc.). Although some spillover of their benefits to neighbouring communities is possible, local public goods are ultimately restricted in space. Like collective goods, local public goods satisfy the exclusion principle.

g) *Public services* is a generic term for the collective goods and the local public goods that will be used in this paper.

[2] *Social property* is a special type of property with distinct legal, social, and economic characteristics that make exploitation impossible. (Exploitation here means: (a) command over the labour of others; and (b) appropriation of non-labour income.) There are two types of social property, collective and individual, the former concerning all members of the society (not just a collection of its members), and the latter pertaining only to family businesses. Here are the defining characteristics of the concept of social property:

a) The *legal rights* of persons disposing of socially-owned commodities and productive assets are:

1. To use, change, or sell commodities, including means of production; and
2. To reap benefits from the use of productive assets (usufruct); while their fundamental *legal obligation* is that:
3. The value of productive assets must not be diminished, whatever the source of original finance.

b) The *social rights* of members of society with respect to socially-owned productive assets are:

4. Every member of society has the right to work;
5. Every member of society has the right to compete for any job, according to his personal capability (consistent with the specifications of the job).
6. Every member of society has the right to participate in management on equal terms.

c) The fundamental *economic right and obligation* of producers under a system of social ownership is that:

7. Every member of society derives economic benefits exclusively from his work, and none from property.

All the rights and obligations enumerated apply to the collective type of social property (e. g., labour-managed firms, schools, universities, hospitals, theatres, etc.). With the exception of the legal obligation (3) and the social rights (4) and (5), they also apply to the individual type of social property (e. g., small-scale private businesses, private restaurants, galleries, etc.). For details see Horvat [5], pp. 235—239.

buted into the wage fund, the investment fund, and the so-called collective consumption fund.[3]

From the economic point of view, the difference between these two types of firms lies in the fact that, unlike consumer goods, public goods are not exchanged in an ordinary market (hence the term *non-exchange goods).* Differential access to educational, health, or judicial establishments would violate the three fundamental dimensions of equality in Yugoslav self-governing society,[4] and public goods are exempt from exchange relationships and distributed differently from the purchasing power of individuals. A special system of so-called self-managing communities of interest *(SIZs* henceforth) has been devised for this purpose.[5] The SIZ is an organization

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[3] The purpose of the collective consumption fund is to provide the employees with some collective goods, such as continuing education, tickets for cultural and artistic events, sports and recreation, as well as with some consumer goods, like hot meals, summer and winter vacation programmes, housing, etc.

[4] The three dimensions of equality that serve as basis for the Yugoslav Constitution are:

a) *Equality of producers,* which implies social ownership of productive capital and workers' management in all firms;

b) *Equality of consumers,* which implies distribution according to work performed, and distribution according to need (which is the fundamental principle for distribution of public goods);

c) *Equality of citizens,* which implies socialist democracy — deconcentration and decentralization of political power with free and equal participation of all the citizens in political life.

These aspects of equality are not explicitly stated in the Yugoslav Constitution of 1974 in this form, which is due to Horvat [5], pp. 229—231, but they are implicitly present in it.

[5] The system of financing of public expenditures in Yugoslavia can be divided in three subsystems:

a) The financing of general social needs;

b) The financing of collective needs in the field of social services;

c) The financing of collective needs in the sphere of material reproduction.

General social needs are the operation of government agencies and institutions, national defence and financing of the development of the less developed areas. They are financed from federal, republic, and communal budgets. The funds for these budgets are provided through enterprise income taxes (C, R, F), personal income taxes (C), sales taxes (R, F), duties (R, F), tariffs (F), and real estate taxes and income from real estate (C), where C refers to communal, R to republic, and F to federal budget.

Collective needs in the field of social services can be divided in two groups:

a) Social activities (health, education, culture, science, physical culture);

b) Social welfare (retirement and disability insurance, child welfare, employment, social security, and housing).

They are financed in three different ways:

1. Through self-managing communities of interest;
2. Through direct exchange between the users and the providers of these services;
3. Through voluntary contributions of citizens, voted for in a referendum.

Thus, the main source of funds for these activities are the enterprise and personal incomes.

gathering, on one side, the representatives of business firms and citizens (the *users* of public services), and, on the other side, the representatives of educational, medical, cultural, scientific and similar establishments (the *providers* of public services). These two parties negotiate in the SIZ on the scope and contents of the output of public services, and on the prices that business firms and citizens will pay for these services. These prices are typically different, which means that citizens and business firms finance different proportions of the output of public services.[6] For goods that satisfy the exclusion principle (except education and medical care), the prices that consumers pay are normal market prices (e. g., the box-office price of a ticket for a concert or a theatre performance), while the prices that business firms pay are usually specified in terms of percentage points of a given accounting category (mainly the net revenue after taxes at the end of the previous business year), that have to be transferred to the proper SIZ each month.[7] The SIZ then distributes these funds to the firms that provide the public service in question according to current production plan negotiated. All decisions on output and prices must be reached by a consensus of parties negotiating in the SIZ. Once reached, such decisions become contractual obligations for the parties and the period of time involved. Thus, business firms and citizens finance the production of public services in a kind of a quasi-market, where they pay different prices for the same services. If we interepret these prices as taxes, the SIZ comes very close to the Lindahl theory of determination of public expenditures.[8] The purpose of this paper is to present a model describing this unique system of financing of public services, to specify the Lindahl equi-

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Collective needs in the sphere of material reproduction are public utilities, public transport and communications, water and energy supplies, communal activities, loans for the development of the under-developed areas, and disaster reliefs. They are entirely financed through the self-managing communities of interest. The sources of funds for their financing are secured loans, fees, and citizens' contributions, voted for in a referendum.

For details of financing public services in Yugoslavia see Jurković [8], and Jurković, Jasić, and Lang [9]. In terms of the definitions from Footnote 1, »general social needs« are pure public goods, »collective needs in the field of social services« are collective goods, and »collective needs in the sphere of material reproduction« have the character of local public goods.

[6] Although the price that citizens pay is sometimes higher (e. g., tickets for various cultural events), the business firms usually end up paying more. Good examples are tuition for continuing education, fees for recreational activities, and, in particular, the medical care and education, where the price the citizens pay is zero.

[7] The taxes in question are enterprise income taxes (in case of socially-owned firms), or personal income taxes (in case of privately-owned businesses) that are used for financing of the general social needs (i. e., pure public goods).

[8] The relevant section of Lindahl's original work *Die Gerechtigkeit der Besteuerung* [Lund 1919] was published in English in *Classics in the Theory of Public Finance*, R. A. Musgrave and A. T. Peacock, eds. [London: Macmillan, 1958], pp. 168—176. In this paper we shall use Johansen's interpretation of Lindahl's theory (Johansen [6] and [7], pp. 129—140).

librium for such an economy, and to consider alternative ways of financing the production of these services. These topics will be dealt with in Parts II and III of the paper, while in the Conclusion we critically evaluate some assumptions of our model, and give some suggestions for future work on this subject.

II A MODEL OF THE PUBLIC SERVICES SECTOR IN YUGOSLAVIA

A. *The concept of Lindahl equilibrium*

In 1919 the Swedish economist Erik Lindahl proposed a solution to the problem of determining simultaneously the amount of public expenditures and the distribution of the corresponding tax burden. Since then, the concept of Lindahl equilibrium has developed into an analytical benchmark in the theory of public expenditures, in many respects similar to that of competitive equilibrium in private ownership economies. In this section we shall briefly present the main ideas of the Lindahl theory and indicate some of its later improvements.

The problem of a simultaneous determination of the amount of public expenditures, and the distribution of the tax burden for their financing can be posed in the following form. Given two groups of people, H and F, buying one private good, Q_1, and one public good, Q_2, with their fixed incomes, I^h and I^f, and given the prices q and p of the private and the public good, what is the amount of public expenditures pQ_2, and the fraction of the total cost of production of the public good (t for H, $1 - t$ for F), that both groups H and F will accept as desirable? The social environment of this problem can be an agency where groups H and F negotiate on their own, or a planning bureau, where social planners have accurate information on preferences of groups H and F. t and $(1 - t)$ — the so-called *Lindahl prices* — can be regarded as taxes that H and F must pay to the provider of the public good, either directly, or through an organization such as the government. Thus, the prices that H and F actually pay for the public good are t and $(1 - t)$, not q. q can be better characterized as the cost of producing one unit of Q_2, the cost that must be supported by H and F through taxes.

Without loss of generality, we let $q = p = 1$, so that Q_1 and Q_2 denote expenditures on two goods. The agents' budget constraints then become:

$$Q_1^h + t\,Q_2 = I^h, \qquad Q_1^f + (1 - t)\,Q_2 = I^f, \tag{I}$$

which gives rise to the national income identity $Q_1 + Q_2 = I$, where

$$Q_1 = Q_1^h + Q_1^f, \qquad I = I^h + I^f.$$

Assuming that the preferences of H and F over private and public goods can be represented by well-behaved utility functions, such as:

$$U^h = S^h(Q_1^h, Q_2), \qquad U^f = S^f(Q_1^f Q_2), \qquad \text{(II)}$$

with positive first-order, and negative second-order partial derivatives in both arguments, this problem can be analysed graphically. *Figure 1* displays a preference map of group H over expenditures on private and public goods. We drew two indifference curves, and two alternative budget lines, the steeper one corresponding to the higher value of t (t closer to 1).

For each value of t, group H prefers that combination of expenditures on the private and the public good which maximizes its utility. This occurs at points such as A and B, or, in general, at points on the curve Z — Z', where the budget lines are tangential to the highest attainable indifference curves. The left-end point of this curve is reached when t = 1, i. e., when H bears the entire cost of producing the public good. Since we assumed regular preferences, there is no right-end point of the curve Z — Z', i. e., if the public good were provided free of any charge (t = 0), we should expect H to desire an unlimited amount of it.

The relationship between the share of the tax burden t, and the expenditures on the public good Q_2, can be better analysed in Figure 2. For this purpose, we substituted Q_1^h and Q_1^f from (I) into (II), thus making U^h and U^f functions of t and Q_2:

$$U^h = S^h(I^h - tQ_2, Q_2), \qquad U^f = S^f(I^f - (1 - t)\,Q_2, Q_2) \qquad \text{(III)}$$

In *Figure 2* we drew a family of indifference curves for H in the (t — Q_2) plane. They correspond to the indifference curves in the (Q_1^h — Q_2) plane from Figure 1. The same is true of the points A

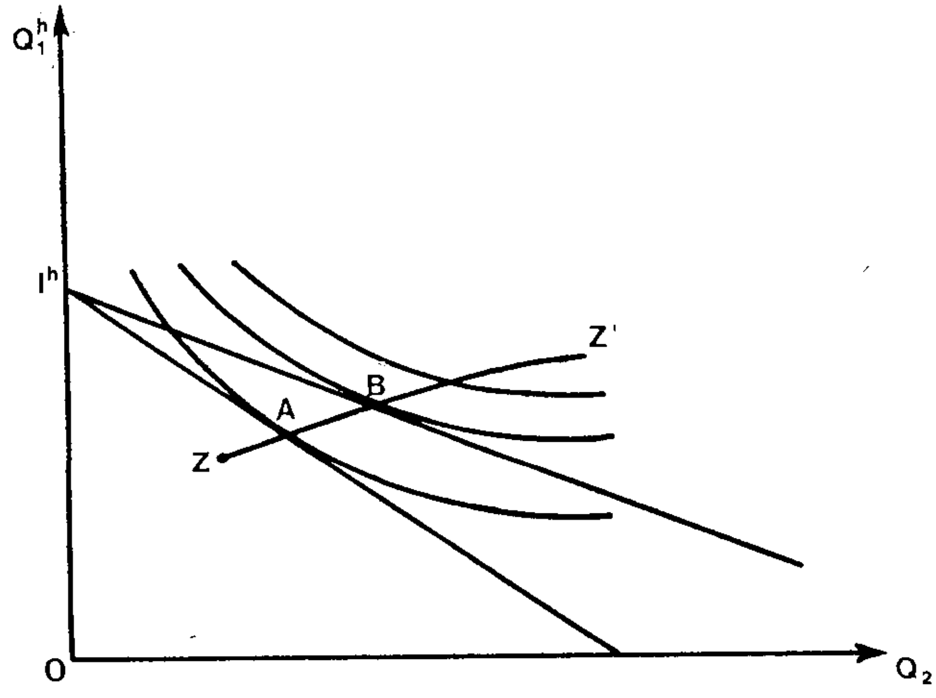

Figure 1: Indifference curves and alternative budget lines in a Q_1^h,Q_2) — diagram.

and B, and the curve Z — Z′, whose similarity to an ordinary, downward sloping demand curve now becomes more apparent. Note that we assume that people in group H will want to spend a positive amount on Q_2 even if they have to bear the entire cost of its production, i. e., even if $t = 1$. Note also that a horizontal movement in Figure 2 corresponds to a movement along the budget line in Figure 1.

We can construct an analogous diagram for group F, but since (1 — t) plays the same role for F as t for H, a more illuminating

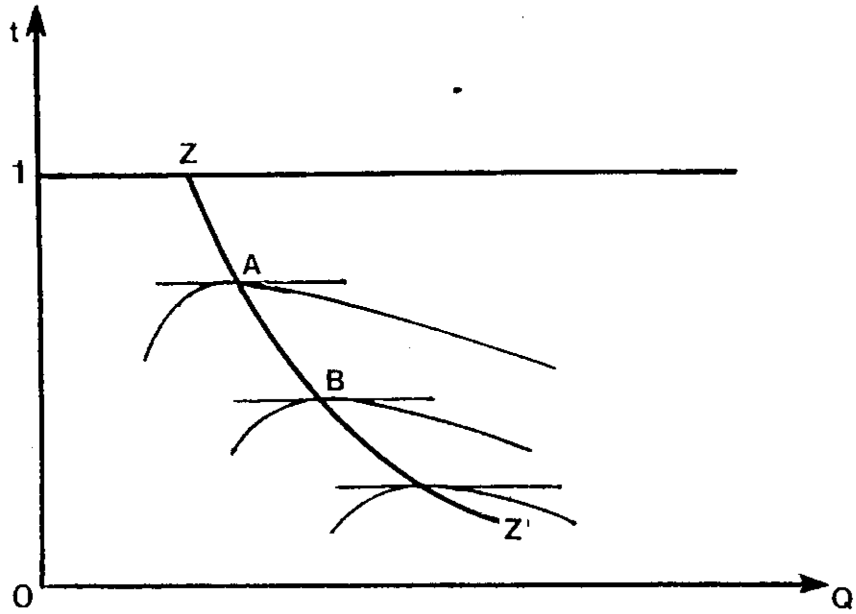

Figure II: Indifference curves for group H in a (t, Q_2) — diagram.

exercise is to combine the two maps in the same diagram. This is done in *Figure 3,* where full lines represent the indifference curves of H, and broken lines the indifference curves of F. The curve C — C′ is the group F analogue of the curve Z — Z′: it shows the amount of expenditures on the public good preferred by F at different values of t. The *Lindahl solution* to the problem of determining Q_2 and t simultaneously is given by point E at the intersection of the curves Z — Z′ and C — C′. This is the only point where both H and F agree on the amount of public expenditures Q_2^* for the given value of t^*. In this respect, the Lindahl solution is very similar to determination of price in an ordinary market, in which both suppliers and demanders consider the price as given. (In our example, the demanders would be the members of group H, while the suppliers would be the members of group F.) The most significant difference from the concept of competitive equilibrium is that in Lindahl equilibrium we observe two different prices for the same good — t and (1 — t) — or, in an economy with n agents, n different prices instead of just one.

If we started with a value of $t_1 > t^*$, where H prefers $Q_2 < Q_2^*$, and F prefers $Q_2 > Q_2^*$, we should expect that group F, in order to induce group H to increase its consumption of the public good, will

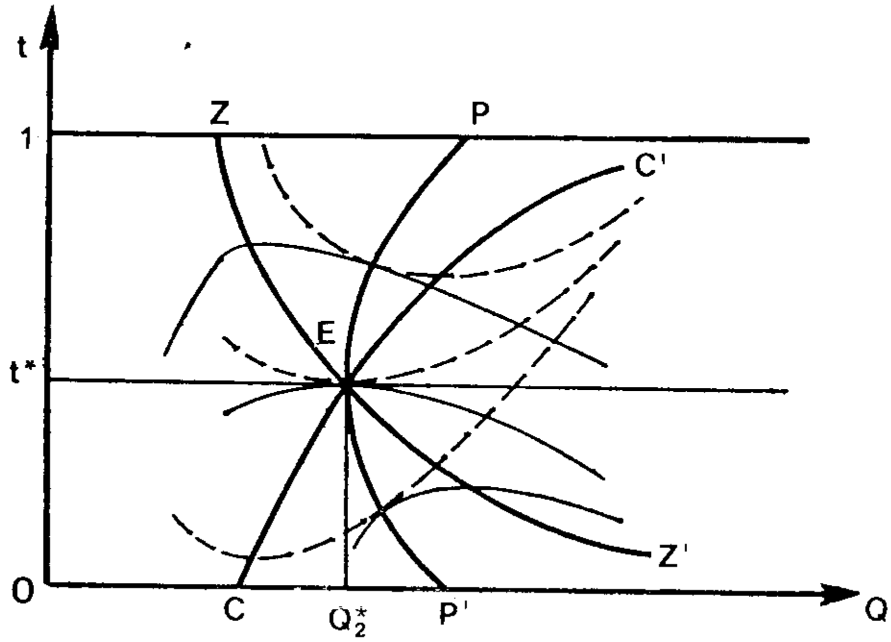

Figure III: The »Lindahl diagram« supplemented with the indifference curves.

propose to bear a larger part of public expenditures. This will cause t to fall, and, eventually, one might hope to reach the equilibrium point E. Such a »tâtonnement« process seems plausible for the case $t_2 < t^*$ as well. Thus, there seem to exist natural tendencies pushing the distribution ratio t towards its equilibrium value t*, assuming, of course, that the two groups H and F have equal power and ability to defend their own interests.[9]

Next, we turn to welfare considerations of the Lindahl solution. The natural question to ask here is: given that the Lindahl solution is so similar to competitive equilibrium, will it also be Pareto optimal under equivalent regulatory conditions? In other words, is there a locus of points (t, Q_2) such that it is impossible to move from one of them in hope of increasing the utility of one group, and at the same time avoid a decrease in the utility of another group? In Figure 3 the locus of such points is described by the curve P — P′, the so-called *contract curve,* derived by tracing out the points of tangency between the two families of indifference curves. We see that point E lies on this curve, which means that the Lindahl solution is Pareto optimal. Naturally, this statement requires a rigorous proof, which is beyond the scope of this section.[10] We shall note, however, that there is no guarantee that the Lindahl solution, although Pareto optimal, will produce maximum social welfare, in the sense of maximizing some social welfare function $W(U^h, U^f)$.

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[9] Readers with some doubts about this mechanism may find a support to their opinion in Johansen [7], pp. 350—353.

[10] Interested readers may consult papers by Foley [4], and Milleron [11].

The maximum of $W(U^h, U^f)$ might be anywhere along the curve P — P′, and only incidentally at point E itself.[11]. For this reason, Lindahl considered that a solution such as E would be a good solution only if the income distribution (I^h, I^f) before the parties concerned were subject to taxation for covering public expenditures was accepted as being equitable. If (I^h, I^f) cannot be considered as equitable, then the problem of taxation and public expenditure requires two steps: first, a pure redistribution of income between the two groups takes place; and then Q_2 and t are determined as in Figure 3. But even if practiced, such a procedure could not entirely satisfy the two parties, neither with respect to the redistribution of income, nor with respect to the ensuing allocations of t and Q_2.[12]

Another difficulty with the Lindahl solution is its relation to the concept of the *core* of an economy. An allocation (of goods among individuals) is said to be in the core if no coalition of individuals can together propose an alternative allocation of its *own* resources that makes at least one member better off and no member worse off — they cannot in this sense improve upon the allocation.[13] There are two well-known theorems relating the core and the competitive equilibrium of regular exchange economies (see Debreu and Scarf [2]):

I Every competitive equilibrium is in the core.

II As the number of traders increases, the set of core allocations shrinks the set of competitive equilibria.

We should expect that equivalent propositions hold for economies with public goods, i. e., that every Lindahl equilibrium belongs to the core, and that the core of an economy with public goods shrinks to the set of Lindahl equilibria as the number of agents increases. But although the first proposition holds under certain conditions (see Foley [4] and Milleron [11]), there is no equivalent for the second proposition. Muench [12] constructed an example of the economy with a continuum of agents where the Lindahl equilibrium is unique, but the core is very large. The intuition behind this result is that, in an economy with public goods, the coalition must be able to make its members better off even if the members outside the coalition decide to produce *no* public goods. For this reason, larger coalitions will more easily improve upon the proposed allocation than small coalitions, which means that the core of an economy with public goods is likely to be bigger than the core of a regular exchange economy.

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[11] This follows from the structure of the social welfare problem, which poses essentially the same constraints as the problem of finding. Pareto optimal points.

[12] That the Lindahl solution is not free from other equity defects is shown in Johansen [7], pp. 136—138. The case in point is the one in which one group regards Q_2 as an inferior good. Under certain conditions, the group with lower income can end up paying a larger portion of public expenditures, even if it considers public good as inferior.

[13] Atkinson and Stiglitz [1], p. 510. In the standard Edgeworth box diagram, the core is a segment of the contract curve between the indifference curves through the initial endowment.

As a consequence, the practical appeal of the concept of Lindahl equilibrium might seem somewhat lesser, since one cannot argue that, irrespective of the mechanism for allocation of public goods, any allocation that arose in free trade and production among the agents could have been achieved by the Lindahl price mechanism. Nevertheless, there is a growing body of literature — excellently surveyed by Tulkens [15] — that still finds a considerable appeal in the Lindahl theory, especially with respect to designing dynamic processes for bringing the economy from its current state, through a sequence of intermediate states, to the state such as described by the Lindahl equilibrium. In this sense, the model that we are going to present on the following pages might be considered as an additional proof of the relevance of the Lindahl theory of determination of public expenditures.

B. *Description of the model*

The agents in this economy are a representative *household*, two firms — one producing consumer goods, called the *business firm*, and one producing collective goods, called the *public services firm* — and one institution, the self-managing community of interest, called the *SIZ*.

Household members work in both firms and purchase consumer and collective goods with the income they earn. The business firm uses labour and collective good as inputs in production of the consumer good, while the public services firm produces its output with the help of labour and the consumer good. Both firms are socially owned, which in the context of this model means that all the net revenue (the »profit«) is distributed to the workers. Consumer good is bought and sold in the market, while the output of the collective good is determind in the SIZ and sold directly to the household and the business firm at prices p_1 and p_2, respectively. All three agents are represented in the SIZ, which serves as a quasi-market for the collective good.

i) THE HOUSEHOLD'S OPTIMIZATION PROBLEM

The household chooses how much to work, and how much of the consumer and collective good to purchase so as to maximize its utility subject to its budget constraint. The specific numerical form of the household's problem is:

$$\max_{L,\, Q_1^h\, Q_2} U\,(L^\circ - L,\, Q_1^h,\, Q_2) = (L^\circ - L)\, Q_1^h\, Q_2 \tag{1}$$

subject to:

$$(w + \Delta w)\, L = q\, Q_1^h + p_1\, Q_2 \tag{2}$$

where:

- L° = 24 hours times the number of employed members of the household
- L = total number of hours the household's members worked in a given period
- Q_1^h = household's consumption of the consumer good
- Q_2 = consumption of the collective good
- w = accounting wage rate in the labour-managed firm
- Δw = variable wage rate in the labour-managed firm
- q = market price of a unit of the consumer good
- p_1 = unit price paid by the household for the collective good, determined in the SIZ

The term Δw comes from the theory of the labour-managed firm. »At the beginning of an accounting period, the workers' council sets the aspiration level of personal income to be achieved. The aspiration income consists of the last period's income or some standard personal income (w) and a change — normally an addition — to be achieved in the current period (Δw). The aspiration income is a function of: a) expected sales; b) incomes in other firms; c) incomes in the preceding and the previous years; d) labour productivity; e) cost of living; f) taxation policy; g) longer-run prospects for income and employment expansion. Since personal income is fixed, what remains to be done is to maximize the residual surplus«:

$$\pi_1 = q\, Q_1 - (w + \Delta w)\, L_1 - p_2\, Q_2$$

$$\pi_2 = (p_1 + p_2)\, Q_2 - (w + \Delta w)\, L_2 - q\, Q_1^f$$

»At the end of the period, aspiration income is revised upward or downward depending on the success of the firm. Thus, we must distinguish between the two types of income: accounting income ($w + \Delta w$) and the personal income actually paid ($w + \Delta w + \Delta w'$). Only the former enters the objective function; for all practical purposes it performs the allocation role of the wage rate — without, however, being a wage rate.«[14]

Note that both the accounting wage rate (w), and the variable wage rate (Δw) are different from firm to firm. However, as a consequence of utility maximization by the households, and the net revenue maximization by the firms, in equilibrium it must be true that:

$$w_1 + \Delta w_1 + \Pi_1 = w_2 + \Delta w_2 + \Pi_2 = w + \Delta w$$

which justifies our usage of the symbol ($w + \Delta w$) in all three maximization problems.

[14] Quoted from Horvat [5], pp. 342—343.

ii) THE BUSINES FIRM'S OPTIMIZATION PROBLEM

The business firm combines labour and consumer good — e. g., continuing education for its employees — so as to maximize its net revenue. The specific numerical form of this problem is:

$$\max_{L_1, Q_2} \Pi_1 = qQ_1 - (w+\Delta w)\ L_1 - p_2Q_2 \tag{3}$$

subject to:

$$Q_1 = (L_1)^a (Q_2)^{1-a}, \qquad 0 < a < 1, \tag{4}$$

where:

Q_1 = total output of the consumer good, $Q_1 = Q_1^h + Q_1^f$
L_1 = total volume of hours of work
p_2 = unit price paid by the business firm for the collective good, determined in the SIZ

iii) OPTIMIZATION PROBLEM OF THE PUBLIC SERVICES FIRM

Inputs in production of the collective good are labour and the consumer good — e. g., the books — and the objective of the public services firm is to maximize its net revenue. The specific numerical form of this problem is:

$$\max_{L_2, Q_1^f} \Pi_2 = (p_1+p_2)\ Q_2 - (w+\Delta w)\ L_2 - qQ_1^f \tag{5}$$

subject to:

$$Q_2 = (L_2)^b (Q_1^f)^{1-b}, \qquad 0 < b < 1, \tag{6}$$

where:

Q_2 = total output of collective good
L_2 = total volume of hours of work
Q_1^f = input of consumer good in production of collective good

iv) THE ECONOMY'S PHYSICAL CONSTRAINTS

In order to assure that a solution to the above optimization problems exists, we impose the following physical constraints:

1. $L_1 + L_2 = L$ (7)

which means that the total supply of labour (L) must be equal to the demands for labour by the business firm (L_1) and by the public services firm (L_2).

2. $Q_1^h + Q_1^f = Q_1$ (8)

that is, the aggregate demand for the consumer good must be equal to the total supply of the consumer good.

3. $Q_2^h = Q_2^f = Q_2$ (9)

which means that the amounts of the collective good that are consumed by the household (Q_2^h) and the business firm (Q_2^f) must be equal to each other and to the total amount of the collective good produced (Q_2).

v) THE ROLE OF THE SIZ

The self-managing community of interest — SIZ — collects the expenditures of the business firm on the collective good, $p_2 Q_2$, and distributes them to the public services firm. For simplicity, we shall assume that the cost of this operation is zero. The SIZ also serves as a negotiating table for the three parties involved in determining the prices and the output of the collective good. These parties are:

(1) the representatives of the public services firm;
(2) the representatives of the business firm; and
(3) the representatives of the households.

As a consequence, there are two distinct, though closely related aspects of negotiations in SIZ:

First, parties (2) and (3) — the users of public services — negotiate with party (1) — the provider of the public services — on the magnitudes of the variables ($p_1 + p_2$) and Q_2 — the total cost of production of the collective good, and the quantity of the collective good that ought to be produced.

Second, party (2) negotiates with party (3) on the distribution of the total cost of financing of public services between the household and the business firm, i. e., they negotiate on p_1 and p_2.

As users of public services, parties (2) and (3) have common interests *vis-á-vis* the party (1), but as parties paying different prices for the collective good, they could *a priori* have different interests. However, we may note that in a labour-managed economy this kind of differences in interests is not likely to complicate the negotiating process. The higher the relative price p_2/p_1, the smaller will be the wage fund of the business firm. A lower p_2/p_1 would imply a larger wage fund of the business firm, but a higher expenditure on the collective good on the part of the household. Since under labour-management the same persons distribute both the firm's income and the household's income, the representatives of the households (typically organized through local communities) are not likely to insist on too large a price differential in their favour. The same holds true for the representatives of the business firms, who have practically no incentive to shift the burden of financing of public expenditures to their *alter ego* — the households.

Concerning the *first aspect* of SIZ negotiations, there are several regulatory constraints that guarantee a socially desirable outcome of negotiations in almost all conceivable situations. They are:

1. The agenda for the SIZ meetings is set by a committee consisting of an equal number of representatives of the three negotiating parties.

2. Representatives of all three parties have *imperative mandates* from the bases that elected them. These imperative mandates ought to be understood as a region (or an interval) in the set of admissible votes, not as a single point in this set. What is admissible is determined through policy (e. g., compulsory ten-year education), goals set by the social plans or the production plans, as well as by the current state of the demand for public services, as revealed through continual communication between the electorate and their representatives.

3. SIZ representatives can vote for a given proposal or submit a new proposal only after consulting their electorate. This is the so-called *public discussion* requirement, now widely practiced at all levels of government in Yugoslavia. All proposals submitted in the SIZ must be presented for discussion in business firms, the local communities where the households live, and public services firms. If a proposal gets approval in the public discussion, then it is ready for voting in the SIZ. If it is amended in public discussion, a proposal must be renegotiated in the SIZ.

4. All relevant information is available to the negotiating parties.

5. All decisions on Q_2, p_1, and p_2 must be reached by *consensus*.

These five requirements — in particular the imperative mandate and the mandatory public discussion — exclude *individual* utility of the SIZ representatives as the dominating criterion in negotiations about the prices and the output of the public services. We must note, however, that making unanimity a necessary condition for decision-making in the SIZ might be undersirable with respect to *group* interests of negotiators. In most situations arising in SIZ negotiations, each representative will feel his group to be so insignificant a part of the whole process that he will direct his attention towards issues he estimates will benefit him, without considering any of the repercussions of his choice. It is conceivable, however, that in the tâtonnement process induced by the consensus requirement, a situation might arise in which the representatives of some group(s) feel the temptation to withhold their vote from proposals that benefit their group a little, in order to blackmail the representatives of other groups into adopting the proposal which benefits them a lot.

Such concentrations of power are unlikely in a labour-managed economy for several reasons: *first*, we already showed that in a situation where the same persons distribute both the firm's and the household's incomes, there are no incentives for negotiators to throw the weight of supporting the public expenditures onto their *alter ego*. *Second*, the self-managing communities of interest are organized both on a functional and on a territorial basis (e. g., the SIZ for culture in city Z), so the degree of interest compatibility among the users and providers of public services is likely to be very high, and the number of major conflicts very low. There is no strong reason to believe that the representatives of the three parties will not be public-spirited and willing to compromise in such a situation. *Third*, there are often no economically or legally feasible alternatives for provision of public goods (e. g., the law in Yugoslavia does not allow opening of private schools or colleges, and no major expansion of private medical care is yet feasible), so the dissenting coalitions would have to be very large, and the cost of their formation is likely to outweigh the benefits they could provide to their members. *Finally*, public projects directly affecting welfare of a large number of members of some communities are typically financed by the so-called *self-contributions*, voted for in a referendum.[15] The proposal for a referendum is usually worked out in the appropriate SIZ, and has to go through a thorough public discussion long before it is submitted for the final vote. It is hard to imagine that any particular group could successfully manage this whole process in its own interest.

To summarize, the decision-making rules in the SIZ appear to be sufficiently elaborate to eliminate the importance of either individual or group interests in deciding about the public expenditures. As economists, we may note that the SIZ plays a role in many respects similar to that of a *Walrasian auctioneer:* as prices get announced, the three parties adjust their supplies and demands until an equilibrium quantity is reached. Or, equivalently, given that some amount of public goods must be produced and consumed (e. g., compulsory education), the three parties negotiate on prices until they contrive a mutually satisfactory agreement. Once the equilibrium prices and quantities are determined, the parties sign contracts on future deliveries and payments, and the market closes until the next year. Hence, *before* the negotiations in the SIZ start, prices and quantities of public goods are in the nature of *stochastic variables*. *After* the end of negotiations they become *parameters*, just as in an ordinary competitive market where buyers and sellers consider prices as given. This has some extremely important consequences for the behaviour of the households and firms, which, in turn, makes financing of public expenditures through the SIZ superior to any other form of taxation commonly practiced. We shall return to this subject in Part III, where

[15] Practically each commune or city in Yugoslavia presently supports at least one of such self-contributions. The funds for self-contributions approved in a referendum are withheld from the income of all employees, and the objects financed in this way range from the water supply systems or schools, to hospitals and facilities for the Olympic games.

we shall examine alternative forms of financing of public expenditures.

C. *A Lindahl equilibrium for the labour-managed economy*

i) DEFINITION

A Lindahl equilibrium for the labour-managed economy described on the previous pages is defined as a set of allocations L^*, L_1^*, L_2^*, Q_1^{h*}, Q_1^{f*} Q_2^*, and prices $(w^* + \Delta w^*)$, q^*, p_1^*, p_2^*, such that the following three conditions are satisfied:

1. (Feasibility)

a) $L_1^* + L_2^* \leq L$

b) $Q_1^{h*} + Q_1^{f*} \leq Q_1$

c) $Q_2^* \leq Q_2$

2. (Net revenue maximization)

Given the prices $(w^* + \Delta w^*)$, q^*, and p_2^*, the input-output plans chosen by the firms solve their maximization problems:

d) $q^* Q_1^* - (w^* + \Delta w^*) L_1^* - p_2^* Q_2^* \geq q^* Q_1' -$
$- (w^* + \Delta w^*) L_1' - p_2^* Q_2'$

e) $(p_1^* + p_2^*) Q_2^* - (w^* + \Delta w^*) L_2^* - q^* Q_1^{f*} \geq (p_1^* + p_2^*) Q_2' -$
$- (w^* + \Delta w^*) L_2' - q^* Q_1^{f\prime}$

for all L_1', L_2', $Q_1^{f\prime}$,Q_2' such that:

f) $L_1' + L_2' \leq L$

g) $Q_1^{h\prime} + Q_1^{f\prime} \leq Q_1$

h) $Q_2' \leq Q_2$

3. (Utility maximization)

Given the prices $(w^* + \Delta w^*)$, q^*, and p_1^*, the allocations of work-time, consumer goods, and the collective good chosen by the household solve its optimization problem:

i) *If* $U(L', Q_1^{h\prime}, Q_2') \geq U(L^*, Q_1^{h*}, Q_2^*)$,
then $q^* Q_1^{h\prime} + p_1^* Q_2' > q^* Q_1^{h*} + p_1^* Q_2^*$
for all $L' \leq L$, $Q_1^{h\prime} + Q_1^f \leq Q_1$, $Q_2' \leq Q_2$.

As we can see, this definition of Lindahl equilibrium includes: i) feasibility of the production and the consumption allocations (L, Q_1, and Q_2 denote available labour, and technologically attainable outputs in the short run); ii) net revenue maximization by the firms; and iii) utility maximization by the households; as equilibrium conditions. It also specifies different prices for the public good paid by the household and the business firm. Our next task is to calculate the equilibrium allocations and prices for the above model of the labour-managed economy. For this purpose we shall define three new variables — the relative prices $q/(w + \Delta w)$, $p_1/(w + \Delta w)$, and $p_2/(w + \Delta w)$ — and denote them r_1, r_2, and r_3, respectively.

ii) SOLVING THE AGENT'S OPTIMIZATION PROBLEMS

a) The household

We start by defining the Lagrangean function for the household's problem:

$$\mathcal{L}\ (L, Q_1^h, Q_2, \mu) = U\ (L^\circ - L,\ Q_1^h,\ Q_2) + \\ + \mu\ [(w + \Delta w)\ L - q\ Q_1^h - p_1\ Q_2] \qquad (10)$$

Since the utility function is concave in L, Q_1^h, and Q_2, and the constraint function is linear, the function $\mathcal{L}$ (.) is also concave, so the first-order conditions are necessary and sufficient for local maximum. These conditions are:

$$\partial \mathcal{L} / \partial L = -Q_1^h Q_2 + \mu(w + \Delta w) = 0 \qquad (11.1)$$

$$\partial \mathcal{L} / \partial Q_1^h = (L^\circ - L)\ Q_2 - \mu q = 0 \qquad (11.2)$$

$$\partial \mathcal{L} / \partial Q_2 = (L^\circ - L)\ Q_1^h - \mu p_1 = 0 \qquad (11.3)$$

$$\partial \mathcal{L} / \partial \mu = (w + \Delta w)\ L - q Q_1^h - p_1 Q_2 = 0 \qquad (11.4)$$

Dividing (11.2) by (11.1), and (11.3) by (11.1) we get the marginal rates of substitution between the consumer good and labour, and between the collective good and labour, respectively. These MRS can be solved for Q_1^h and Q_2:

$$(L^\circ - L)/Q_1^h = q/(w + \Delta w) \qquad Q_1^h = (L^\circ - L)\ /\ r_1 \qquad (12)$$

$$(L^\circ - L)/Q_2 = p_1/(w + \Delta w) \qquad Q_2 = (L^\circ - L)\ /\ r_2 \qquad (13)$$

Dividing (11.4) by $(w + \Delta w)$, and substituting for Q_1^h and Q_2 the equations (12) and (13), we get the household's supply function of labour:

$$L - (L^\circ - L) - (L^\circ - L) = 0 \quad \therefore \quad L = (2/3)\ L^\circ \qquad (14)$$

This result is due to the specific numerical form of the household's utility function. Substituting it back into (12) and (13) yields the household's demand functions for the consumer and the collective goods:

$$Q_1^h = L^\circ/3r_1 \quad (15)$$

$$Q_2 = L^\circ/3r_2 \quad (16)$$

b) The business firm

Next, we solve the problem of the business firm. Its objective function:

$$\Pi_1 = q(L_1)^a (Q_2)^{1-a} - (w + \Delta w) L_1 - p_2Q_2 \quad (3)$$

is concave in the choice variables L_1 and Q_2, so the first-order extremum conditions will be necessary and sufficient for local maximum. These conditions are:

$$\partial\Pi_1/\partial L_1 = qa(Q_1/L_1) - (w + \Delta w) = 0 \quad (17)$$

$$\partial\Pi_1/\partial Q_2 = q(1 - a)\ Q_1/Q_2) - p_2 = 0 \quad (18)$$

Dividing (17) and (18) by $(w + \Delta w)$ we get the business firm's factor demand functions expressed in terms of relative prices:

$$L_1 = ar_1Q_1 \quad (19)$$

$$Q_2 = (1 - a)(r_1/r_3)\, Q_1 \quad (20)$$

Upon dividing (19) by (20) we get the marginal rate of substitution in production between labour and the collective good:

$$L_1/Q_2 = [a/(1 - a)]\ r_3 \quad (21)$$

c) The public services firm

Finally, we solve the problem of the public services firm. Its objective function:

$$\Pi_2 = (p_1 + p_2)(L_2)^b (Q_1^f)^{1-b} - (w + \Delta w) L_2 - qQ_1^f \quad (5)$$

is concave in the choice variables L_2 and Q_1^f, so the first-order conditions for extremum will be necessary and sufficient for local maximum These conditions are:

$$\partial\Pi_2/\partial L_2 = b\ (p_1+p_2)\ (Q_2/L_2) - (w+\Delta w) = 0 \quad (22)$$

$$\partial\Pi_2/\partial Q_1^f - (1 - b)\ (p_1+p_2)\ (Q_2/Q_1^f) - q = 0 \quad (23)$$

Dividing (22) and (23) by $(w + \Delta w)$ we get the public services firm's factor demand functions expressed in terms of relative prices:

$$L_2 = b\,(r_2 + r_3)\,Q_2 \qquad (24)$$

$$Q_1^{t} = (1 - b)\,[(r_2 + r_3)\,/\,r_1]\,Q_2 \qquad (25)$$

Dividing (24) by (25) gives us the marginal rate of substitution in production between labour and the consumer good:

$$(L_2/Q_1^{f}) = [b/(1 - b)]\,r_1 \qquad (26)$$

iii)DETERMINING THE EQUILIBRIUM ALLOCATIONS

a) Equilibrium allocations of labour

We shall first determine the equilibrium allocations of labour between the two firms. We start by equating labour supply and labour demands. From (14), (19), and (24), and the first physical constraint in this economy:

$$(L_1 + L_2) = L \qquad (7)$$

we have that:

$$(2/3)\,L^{\circ} = ar_1Q_1 + b\,(r_1 + r_2)\,Q_2 \qquad (28)$$

Next, we derive the aggregate demand for the consumer good By using the second physical constraint in this economy:

$$Q_1^{h} + Q_1^{f} = Q_1 \qquad (8)$$

and the demand equations for the consumer good (15) and (25), we get:

$$Q_1 = (L^{\circ}/3r_1) + (1 - b)\,[(r_2 + r_3)\,/\,r_1]\,Q_2 \qquad (30)$$

Now we use (30) to substitute out Q_1 from (28). Solving for Q_2 then yields:

$$Q_2 = L^{\circ}\,(2 - a)\,/\,[3(r_2 + r_3)\,(a + b - ab)] \qquad (31)$$

Finally, we substitute (31) into (24), and get the equilibrium demand for labour of the public services firm:

$$L^{*}_2 = L^{\circ}\,b\,(2 - a)\,/\,3\,(a + b - ab) \qquad (32)$$

From the first physical constraint we immediately get the equilibrium allocation of labour to the business firm:

$$L^{*}_1 = L^{\circ}a\,(2 - b)\,/\,3\,(a + b - ab) \qquad (33)$$

b) Equilibrium allocations of consumer and collective goods

Next, we shall determine the equilibrium allocations of the consumer and the collective good. Initially, we shall express them in terms of r_1, and later in terms of parameters only. Equilibrium allocations of the consumer good are those that are demanded by its consumers, the household and the public services firm. Thus,

$$Q_1^{h*} = L^\circ / 3r_1 \qquad \text{[from (15)]} \tag{34}$$

$$Q_1^{f*} = L^\circ (1-b)(2-a) / 3r_1 (a+b-ab) \qquad \text{[from (25) and (31), or from (26) and (32)]} \tag{35}$$

Equilibrium allocation of the collective good is the one that is in equilibrium supplied by the public services firm. Both consumers of the collective good — the household and the business firm — consume it in this quantity. We calculate it by substituting the equilibrium allocations of L_2 and Q_1^f [equations (32) and (35)] into the production function of the public services firm [equation (6)]:

$$Q_2^* = (L^\circ/3)\, r_1^{b-1}\, [b/(1-b)]^b\, [(1-b)(2-a)/(a+b-ab)] \tag{36}$$

iv) DETERMINING THE EQUILIBRIUM PRICES

Finally, we have to determine the equilibrium prices. We shall first calculate the relative prices r_1, r_2, and r_3, and later, by setting the price of the numéraire (labour) equal to one, the prices of the consumer and the collective goods.

a) Equilibrium price of the consumer good

Equilibrium price of the consumer good is the price at which the sum of equilibrium demands for Q_1 equals the equilibrium supply of Q_1. The aggregate demand for Q_1 is derived by summing equations (34) and (35):

$$Q_1^{D*} = Q_1^{h*} + Q_1^{f*} = (L^\circ / 3r_1)\{1 + [(1-b)(2-a)/(a+b-ab)]\} \tag{37}$$

while the aggregate supply of Q_1 is derived by substituting equilibrium allocations of L_1 and Q_2 into the business firm's production function (4):

$$Q_1^{S*} = (L^\circ/3)\,(r_1^{a+b-ab-1})\, a^a b^{b-ab}\, (2-a)^{1-a}\, (2-b)^a\, (1-b)^{1-a-b+ab}\, (a+b-ab)^{-1} \tag{38}$$

Equating (37) with (38) and solving for r_1 gives:

$$r_1^* = \{ [1+(2-a)(1-b)/(a+b-ab)]\, a^{-a} b^{ab-b}\, (2-a)^{a-1}\, (2-b)^{-a}\, (1-b)^{a+b-ab-1}\, (a+b-ab) \}^{(1/a+b-ab)} \tag{39}$$

(For example, with a = b = 0.5, $r_1 = 2^{(4/3)}$.) This r_1^* can now be substituted back into equations (34) — (36) to get the equilibrium consumption allocations. For this purpose we shall write:

$$r_1^* = [(1 + D)E]^z \qquad (39.1)$$

where: $D = (2 - a)(1 - b) / (a + b - ab)$

$$E = a^{-a} b^{ab-b} (2 - a)^{a-1} (2 - b)^{-a} (1 - b)^{a+b-ab-1} (a + b - ab)$$

$$z = 1 / (a + b - ab)$$

The equilibrium allocations of the consumer and the collective good are then:

$$Q_1^{h*} = (L^\circ / 3) [(1 + D)E]^{-z} \qquad (34.1)$$

$$Q_1^{f*} = (L^\circ / 3) D [(1 + D)E]^{-z} \qquad (35.1)$$

$$Q_2^* = (L^\circ / 3) BD [(1 + D)E]^{-z(b-1)}, \text{ where } B = [b / (1 - b)]^b \qquad (36.1)$$

and the equilibrium supply of the consumer good is:

$$Q_1^* = (L^\circ / 3) (1 + D)^{1-z} E^{-z} \qquad (37.1)$$

b) Lindahl prices of the collective good

Next, we have to derive the expression for the personalized prices of the collective good. For this purpose, we shall make use of the household's and the business firm's marginal rates of substitution between the collective good and labour [equations (16) and (19), respectively]. As explained in Section A, Lindahl believed that the solution to the problem of just taxation was that the tax should be borne by the consumers of the public goods according to the utility that they derived from consuming an additional unit of the public good, provided that the income distribution before such taxes was accepted as being equitable. In our economy, the consumers of the public good are the household and the business firm, and the way in which they reveal their preferences is by reporting their marginal rates of substitution between labour and the collective good during the negotiations in the SIZ. We chose the marginal rates of substitution between labour and the collective good, rather than between the consumer and the collective good (which is a procedure usually suggested in the literature) because for one of our consumers — the business firm — the consumer good is an output, not an input in "consumption". From equations (16) and (19) we thus get:

$$r_2^* = L^\circ / 3Q_2^* \qquad (16.1)$$

$$r_3^* = [a/(1 - a)] (L_1^*/Q_2^*) \qquad (21.1)$$

By substituting the equilibrium allocations of L_1 (equation 33), Q_1 (equation 37.1), and Q_2 (equation 36) into equations (16.1) and (21.1), we get the following expressions for the Lindahl prices of the collective good:

$$r_2 = (BD)^{-1} [(1 + D)E]^{z(1-b)} \tag{40}$$

$$r_3 = z(1 - a)(2 - b)(BD)^{-1} [(1 + D)E]^{z(1-b)} \tag{41}$$

(Thus, with a = b = 0.5, $r_2 = 2^{(2/3)}$, $r_3 = 2^{(2/3)}$, which is a special case of identical production technologies. With a = 0.5 and b = 0.4, the Lindahl prices are: r_2 = 1.635038 and r_3 = 1.8686149.)

Finally by setting $(w + \Delta w) = 1$, we get $r_1^* = q^*$, $r_2^* = p_1^*$, and $r_3^* = p_2^*$. The Lindahl equilibrium allocations and prices for this economy are then:

$$LE = \{[L^*, Q_1^{h*}, Q_2^*], [L_1^*, Q_1^*, Q_2^*], [L_2^*, Q_1^{f*}, Q_2^*], [w^* + \Delta w^*, q^*, p_1^*, p_2^*]\}$$

$$LE = \{[(2/3)L^\circ, \quad (L^\circ/3)[(1+D)E]^{-z}, \quad (L^\circ/3) BD [(1+D)E]^{z(b-1)}],$$

$$[(L^\circ/3)az(2 - b), \quad (L^\circ/3)(1+D)^{1-z}E^{-z}, \quad (L^\circ/3) BD [(1+D)E]^{z(b-1)}],$$

$$[(L^\circ/3)bz(2 - a), \quad (L^\circ/3) D [(1+D)E]^{-z}, \quad (L^\circ/3) BD [(1+D)E]^{z(b-1)}],$$

$$[1, [(1+D)E]z, \quad (BD)^{-1} [(1+D)E]^{z(1-b)},$$

$$z(1 - a)(2 - b)(BD)^{-1} [1+D)E]^{z(1-b)}]\}$$

This completes our description of Lindahl equilibrium for the labour -managed economy.

III ALTERNATIVE WAYS OF FINANCING PUBLIC SERVICES: A COMPARATIVE ANALYSIS

Now we shall consider three alternative ways of financing of public services. Instead of the SIZ we shall introduce the government, whose role will be to collect various kinds of taxes from the agents, and to distribute them in the form of unit subsidies to the public services firm. For simplicity, we shall assume that the government incurs no cost in this operation. We shall analyse the effects of three taxes: personal income tax (t_1), commodity tax on the consumer good paid by the household (t_2), and the tax on the business firm's wage fund (t_3). t_1, t_2, and t_3 can come in seven different combinations, but there are only three that are of interest to us: (i) the case of personal income tax; (ii) the case of tax on cosumer good combined with the personal income tax; and (iii) the case of tax on consumer good combined with the tax on the business firm's wage fund.

There are several reasons why we chose these three cases for comparative analysis. *First,* they roughly correspond to tax regimes currently prevailing in the United States, Western Europe, and Eastern European countries. Since we shall retain our framework of the labour-managed economy from Part II, the analysis of three new fiscal systems might suggest an answer to the question: which method of financing of public services is most desirable in the labour-managed economy? *Second,* combining the personal income tax with the tax on the firm's wage fund would mean taxing the same source twice, since in the labour-managed firm all net income belongs to workers, and is distributed into personal incomes by the workers proper. *Third,* we omit the analysis of commodity tax, and of the tax on the firm's wage fund, because they rarely figure on their own.

Thus, in sections D, E, and F we shall introduce in our model three new sets of parameters: (t_1), (t_1, t_2), and (t_2, t_3), and display the equilibrium relations for variables L_1, L_2, Q_1^h, Q_1^f, Q_2, q, p_1, p_2, and s, the unit subsidy to the public services firm. In section G we shall compare these relations with the Lindahl equilibrium results from Part II, and discuss the differences between the alternatives for financing of public expenditures.

D. Personal income tax

In presence of the personal income tax, the household's budget constraint must be modified, so that its optimization problem now becomes:

$$\max_{L,Q_1^h,Q_2} U(L^\circ - L, Q_1^h, Q_2) = (L^\circ - L) Q_1^h Q_2 \qquad (1)$$

$$\text{subject to: } (1 - t_1)\,(w + \Delta w)\, L = q Q_1^h + p_1 Q_2 \qquad (2)$$

As in Part II, the first-order extremum conditions for this optimization problem are necessary and sufficient for local maximum.

These conditions can be written in the following form:

$$Q_1^h = (L^\circ - L)\,(1 - t_1)\,/\,r_1 \qquad (12')$$

$$Q_2 = (L^\circ - L)\,(1 - t_1)\,/\,r_2 \qquad (13')$$

$$L = [r_1 Q_1^h + r_2 Q_2]\,/\,(1 - t_1) \qquad (11.4')$$

where: $r_1 = q/(w+\Delta w)$, $r_2 = p_1/(w+\Delta w)$, are the relative prices of the consumer and the collective good in terms of the price of the numéraire.

Due to a specific form of the utility function, the equilibrium supply of labour will not be affected by personal income tax: substitution of equations (12') and (13') — the demand functions for the consumer and the collective good — into equation (11.4') — the household's budget constraint — yields the same labour supply function as in the no-tax case of Section C:

$$L = (2/3)\, L^{\circ} \tag{14}$$

This is certainly the most surprising result of our model so far, since we were intuitively expecting the labour supply to decrease with respect to the no-tax case. It is doubtful, however that this special feature of our utility function jeopardizes any result that will be the subject of comparative analysis in Section G, so we shall consider it as relatively harmless.

The optimization problem and the first-order extremum conditions of the business firm are the same as in Section C, but those of the public services firm are now slightly different. The problem:

$$\max_{L_2, Q_1^f} \; \Pi_2 = (p_1+p_2+s_1)Q_2 - (w+\Delta w)L_2 - qQ_1^f \tag{5}$$

$$\text{subject to: } Q_2 = (L_2)^b\,(Q_1^f)^{1-b}, \quad 0<b<1 \tag{6}$$

gives rise to following first-order conditions:

$$L_2 = b(r_2+r_3+r_4)Q_2 \tag{24'}$$

$$Q_1^f = (1-b)\,[(r_2+r_3+r_4)\,/\,r_1]Q_2 \tag{25'}$$

where: $r_4 = s_1/(w+\Delta w)$. Finally, the government's budget constraint is given by:

$$t_1 L = s_1 Q_2 \tag{43}$$

From the agents' first-order extremum conditions, the economy's physical resource constraints, and the government's budget constraint, we get a system of 12 equations in 12 unknowns: L, L_1, L_2, Q_1, Q_1^h, Q_1^f, Q_2, $w+\Delta w$, q, p_1, p_2, s_1, and four parameters: L°, t_1, a, and b:

$$L = (L^{\circ}/3)\,2 \tag{14}$$

$$Q_1^h = (L^{\circ}/3)\;[(1-t_1)/r_1] \tag{12}$$

$$Q_2 = (L^{\circ}/3)\,[(1-t_1)\,/\,r_2] \tag{13'}$$

$$Q_1 = (L_1)^a\,(Q_2)^{1-a} \tag{4}$$

$$L_1 = ar_1Q_1 \tag{19}$$

$$Q_2 = (1-a)\,(r_1/r_3)\,Q_1 \tag{20}$$

$$Q_2 = (L_2)^b \, (Q_1^f)^{1-b} \tag{6}$$

$$L_2 = b \, (r_2+r_3+r_4) \, Q_2 \tag{24'}$$

$$Q_1^f = (1-b) \; Q_2 \, [(r_2+r_3+r_4)/r_1] \tag{25}$$

$$L = L_1+L_2 \tag{7}$$

$$Q_1 = Q_1^h+Q_2^f \tag{8}$$

$$t_1 L = r_4 Q_2 \tag{43}$$

where: $r_1 = q/(w+\Delta w)$, $r_2 = p_1/(w+\Delta w)$, $r_3 = p_2/(w+\Delta w)$, $r_4 = s_1/(w+\Delta w)$.

We can solve this system by proceeding along the same lines as in Part II, an exercise that will be omitted on this occasion. Instead, we display only the equilibrium relations for the variables of interest, assuming, as in Part II that, labour is the numéraire:

$$L_1^* = (L^\circ/3) \; [2 - zb \; (2 - aT_1)] \tag{44}$$

$$L_2^* = (L^\circ/3) \; zb \; (2 - aT_1) \tag{45}$$

$$Q_1^{h*} = (L^\circ/3) \; (T_1/q^*) \tag{46}$$

$$Q_1^{f*} = (L^\circ/3) \; z \; (1-b) \; (2-aT_1) \; (1/q^*) \tag{47}$$

$$Q_2^* = (L^\circ/3) \; z \; (1-b) \; B \; (2-aT_1) \; (q^*)^{b-1} \tag{48}$$

$$q^* = \{[T_1 + z \, (1-b) \, (2-aT_1)] \, / \, [2 - zb \, (2-aT_1)]^a \; [zb \, (2-aT_1)]^{1-a} \, [b \, / \, (1-b)]^{(b-1)(1-a)}\}^z \tag{49}$$

$$p_1^* = (L^\circ/3) \; (T_1/Q_2^*) \tag{50}$$

$$p_2^* = (L^\circ/3) \; [(1-a) \, / \, a] \; (2/Q_2^*) \tag{51}$$

$$s_1^* = (L^\circ/3) \; (2t_1/Q_2^*) \tag{52}$$

where:

$T_1 = (1 - t_1)$
$z = 1 \, / \, (a + b - ab)$
$B = [b \, / \, (1-b)]^b$

It is imminent from equations (44) — (52) that the introduction of personal income tax has changed equilibrium values of almost all variables in the economy. In Section G we shall examine the magnitude and the direction of these changes in greater detail.

E. Commodity tax and personal income tax

Adding the tax on the consumer good purchased by the household (t_2), further changes the household's budget constraint:

$$(1 - t_1)(w + \Delta w) L = (1 + t_2)\, qQ_1^h + p_1 Q_2 \qquad (2'')$$

The resulting first-order conditions for maximum of utility function are:

$$Q_1^h = (1/r_1)(L^\circ - L)\,[(1 - t_1) / (1 + t_2)] \qquad (12'')$$

$$Q_2 = (1/r_2)(L^\circ - L)(1 - t_1) \qquad (13'')$$

$$(1 - t_1)\, L = (1+t_2)\; r_1 Q_1^h + r_2 Q_2 \qquad (11.4'')$$

Substituting equations (12″) and (13″) into (11.4″) yields the equilibrium labour supply:

$$L = (2/3)\, L^\circ \qquad (14)$$

Hence, neither the introduction of the commodity tax has changed the household's labour supply. Since the taxes are levied with the purpose of subsidizing the production of public services, we assume that the public services firm will not have to pay the commodity tax on its purchases of consumer good. Therefore, its problem is the same as in Section D, the only difference being the structure of the unit subsidy. Namely, the government's budget constraint now reads:

$$(w + \Delta w)\, Lt_1 + qQ_1^h\, t_2 = s_2\, Q_2, \qquad (53)$$

so that the unit subsidy now includes one additional term:

$$r_5 = (1/Q_2)\,[Lt_1 + r_1\, Q_1^h\, t_2]$$

where $r_5 = s_2 / (w + \Delta w)$. As in section D, we shall omit the computational procedure, and present only the equilibrium relations for the variables of interest in terms of the economy's parameters L°, t_1, t_2, a and b:

$$L_1^* = (L^\circ / 3)\,[2 - zb\,(2 - aT_2)] \qquad (54)$$

$$L_2^* = (L^\circ / 3)\, zb\,(2 - aT_2) \qquad (55)$$

$$Q_1^{h*} = (L^\circ / 3)\,(T_2 / q^*) \qquad (56)$$

$$Q_1^{f*} = (L^\circ / 3)\,[z\,(1 - b)\,(2 - aT_2) / q^*] \qquad (57)$$

$$Q_2^* = (L^\circ / 3)\, z\,(1 - b)\,(2 - aT_2)\,(q^*)^{b-1} \qquad (58)$$

$$q^* = \{[T_2 + z\,(1 - b)\,(2 - aT_2)] \,/\, [2 - zb\,(2 - aT_2)]^a \, [zb\,(2 - aT_2)]^{1-a} \, b^b \, (1 - b)^{(1-b)(1-a)}\}^z \qquad (59)$$

$$p_1^* = (L^\circ / 3)\,(T_2 / Q_2^*) \qquad (60)$$

$$p_2^* = (L^\circ / 3)\,(2 / Q_2^*)\,[(1 - a) / a] \qquad (61)$$

$$s_2^* = [(L^\circ / 3)\,2t_1 + q^* Q_1^{h*}\, t_2] / Q_2^* \qquad (62)$$

where:

$$T_2 = (1 - t_1) / (1 + t_2)$$
$$z = 1 / (a + b - ab)$$
$$B = [b / (1 - b)]^b$$

We easily recognize great similarity of this new set of equilibrium relations with that of Section D, a subject to which we shall return in section G.

F. *Tax on the firm's wage fund and commodity tax*

While in the previous two cases the entire tax burden rested on the household, in this case we shall split the tax burden between the household and the business firm: the former will pay tax t_2 on its purchases of the consumer good, while the latter will be assessed tax t_3 on its wage fund. Since t_2 and t_3 are levied in order to subsidize the production of the collective good, we shall not require the public services firm to support any taxes.

With t_2 in the system, the household's budget constraint has this form:

$$(w + \Delta w)\,L = (1 + t_2)\,qQ_1^h + p_1 Q_2 \qquad (2''')$$

Maximizing the utility function (1) with respect to (2''') yields these first-order conditions for local maximum:

$$Q_1^h = (L^\circ - L) / (1 + t_2)\,r_1 \qquad (12''')$$

$$Q_2 = (L^\circ - L) / r_2 \qquad (13''')$$

$$L = (1 + t_2)\,r_1 Q_1^h + r_2 Q_2 \qquad (11.4''')$$

From this set of equations we once again obtain the same expression for equilibrium labour supply of the household:

$$L = (2 / 3)\,L^\circ \qquad (4)$$

Next, we turn to the analysis of the impact of t_3 on the behaviour of the business firm. Its optimization problem now becomes:

$$\max \Pi_1 = qQ_1 - (1+t_3)\ (w+\Delta w)\ L - p_2Q_2 \tag{3'}$$

subject to:

$$Q_1 = (L_1)^a\, (Q_2)^{1-a}, \qquad 0 < a < 1 \tag{4}$$

Solving the first-order conditions for this problem yields:

$$L_1 = a\, r_1\, Q_1 \,/\, (1 + t_3) \tag{19'}$$

$$Q_2 = (1 - a)\, r_1\, Q_1 \,/\, r_3 \tag{20'}$$

Finally, we modify the government's budget constraint:

$$s\, Q_2 = t_2\, q\, Q_1^h + (w + \Delta w)\, L_1\, t_3 \tag{63}$$

The new unit subsidy is therefore:

$$r_6 = [t_2\, r_1\, Q_1^h + t_3\, L_1] \,/\, Q_2,$$

where $r_6 = s_3/(w + \Delta w)$. Since the problem of the public services firm remains unchanged, we are now in a position to display the equilibrium relations for the economy's variables:

$$L_1^* = (L^\circ / 3)\, [2 - kb\, (2T_3 - a)] \tag{64}$$

$$L_2^* = (L^\circ / 3)\, kb\, (2T_3 - a) \tag{65}$$

$$Q_1^{h*} = (L^\circ / 3)\, [1 \,/\, (1 + t_2)\, q^*] \tag{66}$$

$$Q_1^{f*} = (L^\circ / 3)\, k\, (1 - b)\, [(2T_3 - a) \,/\, q^*] \tag{67}$$

$$Q_2^* = (L^\circ / 3)\, k\, (1 - b)\, B\, (2T_3 - a)\, (q^*)^{b-1} \tag{68}$$

$$q^* = \{ [(1 + t_2)^{-1} + k\, (1 - b)\, (2T_3 - a)] \,/\, [2 - kb\, (2T_3 - a)]^a \\ [k\, (1 - b)\, B\, (2T_3 - a)]^{1-a} \}^z \tag{69}$$

$$p_1^* = (L^\circ / 3)\, (1 \,/\, Q_2^*) \tag{70}$$

$$p_2^* = (L^\circ / 3)\, (2 \,/\, Q_2^*)\, (1 + t_3) \tag{71}$$

$$s_3^* = [t_2\, q\, Q_1^{h*} + t_3\, L_1^*] \,/\, Q_2^*$$

where:

$T_3 = (1 + t_2)\, (1 + t_3)$ $\qquad$ $k = (a + b - ab + bt_3)\, (1 + t_2)$

$z = 1 \,/\, (a + b - ab)$ $\qquad$ $B = [b \,/\, (1 - b)]^b$

This completes our analysis of alternative ways of financing of public expenditures. In the next section, we shall compare the equilibria under the income, commodity, and wage fund taxes with the original Lindahl equilibrium.

G. *Comparative analysis*

In Table I we presented the equilibrium values of L_1, L_2, Q_1, Q_1^h, Q_1^f, Q_2, q, p_1, p_2, s, and the household's utility, for arbitrary values of the economy's parameters, for four alternative ways of financing of public expenditures:

(I) the original SIZ financing scheme;
(II) the personal income tax (t_1);
(III) the consumer good tax and the personal income tax (t_2, t_1);
(IV) the tax on the business firm's wage fund and the consumer good tax (t_3, t_2).

Reading the data in Table I enables us to make some interesting observations about the relative rankings of four financing schemes.

One possible criterion of performance in this model is the aggregate physical output of the economy. The ranking according to this criterion would put the SIZ financing scheme in the last place, and the combination of personal income tax and the consumer good tax in the first place. Maximum output is the typical objective of planners in the centrally planned economies, so we shall not consider it as relevant for our model.

If we judge the performance of a particular economy on the basis of aggregate utility it generates for the households — a criterion typically applied in welfare analysis — then financing of public expenditures through a system of self-managing communities of interest represents a Pareto optimum. This result is not surprising, since in Section C we showed that the SIZ solution was a Lindahl equilibrium for the labour-managed economy, while in Section A we mentioned that under certain regulatory conditions, every Lindahl equilibrium was a Pareto optimum.[16] On the other hand, we know

[16] These regularity conditions are primarily concerned with the properties of the consumption set, the consumer preferences, and the production set. The consumption set should be closed, convex, and bounded from below, and there must be no satiation point in it; the preferences ought to be complete, transitive, continuous, and locally non-satiated; the vector of individual endowments must be strictly interior to the consumption set; while the production set must be closed and convex, contain the zero vector of net inputs, and have the properties of irreversibility and impossibility of free production. Under these conditions we can apply the results of Kakutani's fixed point theorem to prove the existence of Lindahl equilibrium. Once we are equipped with the existence result, the optimality of Lindahl equilibrium follows from a version of the first fundamental theorem of welfare economics.

In our model both the utility and the production functions are strictly concave, with positive first-order and negative second-order partial derifrom conventional welfare theory that in the presence of distortionary taxes, the competitive equilibrium inevitably fails to be Pareto optimal. This can be verified on the data from Table I, too. However, we still owe an explanation for the absence of distortionary effects under the SIZ financing scheme.

TABLE I — SIZ vs. *Tax schemes: Direction and magnitude of changes in the economy's variables.*[17]

Tax scheme Variable	SIZ	(t_1)	Δ%	(t_1, t_2)	Δ%	(t_2, t_3)	Δ%
L_1	8	7.73	— 3.3	7.52	— 6.1	7.27	— 9.1
L_2	8	8.27	+ 3.3	8.48	+ 6.1	8.72	+ 9.1
Q_1^h	3.17	2.99	— 5.9	2.82	—11.3	2.87	— 9.6
Q_1^f	3.17	3.43	+ 8.0	3.65	+15.0	3.44	+ 8.5
Q_1	6.35	6.42	+ 1.1	6.47	+ 1.9	6.31	— 0.55
Q_2	5.04	5.32	+ 5.7	5.57	+10.4	5.48	+ 8.8
q	2.52	2.41	— 4.3	2.32	— 7.8	2.53	+ 0.55
p_1	1.59	1.35	—14.8	1.29	—18.5	1.46	— 8.1
p_2	1.59	1.45	— 8.5	1.35	—14.9	1.46	— 8.1
s	—	0.3	—	0.41	+34.8	0.27	—11.7
$U(L^* - L, Q_1^h, Q_2)$	128	127.25	— 0.6	125.41	— 2.0	125.89	— 1.6

$a = b = 0.5$
$t_1 = t_2 = t_3 = 0.1$
$L^\circ = 24$
$w + \Delta w = 1$

As we saw in Section B, financing of couective consumption through the self-managing communites of interest has the property that, *ex ante*, the agents consider the prices of the collective good as *stochastic*, while *ex post* they treat these prices *parametrically*. In this way we avoided distorting the agents' choices: the household and the business firm solved their optimization problems by treating the cost of financing of public services as an ordinary expenditure item

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vatives in all arguments, and strictly convex contour maps in consumption and input spaces, while the endowment of labour is strictly positive, all of which ensures both the existence and optimality of Lindahl equilibrium.

[17] Although we chose the 10 per cent tax rate arbitrarily, we intentionally introduced the asymmetry between the amount of taxes collected under the first (t_1), and the second two schemes (t_1, t_2), and (t_2, t_3). We identified these tax schemes with the current tax regimes in the United States, Western Europe, and Eastern European countries, and since the share of public expenditures in national income is larger for the European countries than for the US, it seemed natural to set the tax receipts under the scheme (t_1) at a lower level than under the other two schemes. As a consequence, the output of public goods is higher under (t_1, t_2) and (t_2, t_3) than under (t_1), which also corresponds to the real-world situation.

in their budget constraints, not as a distorting tax that would modify their marginal rates of substitution. Furthermore, by linking p_2 to the net revenue of the business firm in the previous period, we actually gave to our financing scheme the character of a *lump-sum subsidy*, in the sense that its magnitude was not a function of the current period decisions.

As far as p_1 is concerned, its magnitude is also independent of the current period decisions of the household, since p_1 can be regarded as a function of the household's income in previous periods, the p_2 / p_1 ratio in the past, the demographic and socio-cultural characteristics of the population (age, sex, literacy, general level of education, etc.), and of some institutional factors (primarily the organization of social services), all of which are likely to be fairly stable over time. Thus, neither p_1 is in the nature of a distorting tax. From this point of view, it seems natural that financing of public services through a system of self-managing communities of interest be Pareto superior to financing alternatives involving distortionary taxation.

We can also find a deeper socio-political argument to confirm this hypothesis. It lies in the fact that determining the economic costs that the agents will have to bear in order to be provided with the public sevices is not left over to the operation of an alienating political process, but is the subject of negotiations in which all parties recognize their mutual objectives and constraints. However, we must admit that the operation cost of the self-managing communities of interest be significant. The consensus requirement and the obligatory public discussion of all relevant agreements proposed in the SIZ prolong the procedure of decision-making and may impede the operational flexibility of participants in negotiations. This price of making the public services system more democratic seems to be inevitable in the early stages of operation of such a system. But as the public gets accustomed to the system, it will also learn how to increase its efficiency.

IV CONCLUSION

In this paper, we have presented a model of financing public services in Yugoslavia. The model consisted of one representative household, a firm producing the consumer good, a firm producing the collective good, and an institution called the self-managing community of interest (SIZ), in which the users and the providers of public services negotiated on output and prices of the collective. good. A Lindahl equilibrium for this labour-managed economy was specified, and three alternative schemes for financing public expenditures were compared to this equilibrium. The SIZ financing scheme proved to be Pareto superior to all three tax-financing schemes. This result was primarily due to absence of distortionary effects of Lindahl prices on the agents' choices, which was, in turn, the consequence of a specific nature of the negotiating process in the SIZ, rigged so as to ensure the achievement of Lindahl equilibrium.

The present analysis could be extended in several directions. One would be to enrich the structure of the model. For example, an explicit cost of negotiating in the SIZ could be specified, and the optimization problems could be generalized so as to avoid some specific results of Cobb-Douglas production and utility functions we encountered in this paper. Next, we could study the model in a dynamic framework, in which the dependence of the prices of the collective good on lagged variables would affect the current decisions of the agents, and thereby the negotiating process in the SIZ. Finally, we could study this model in a stochastic framework, especially with respect to determination of the prices and the output of the collective good. These new directions of analysis would, hopefully, strengthen many conclusions reached thus far, and confirm our belief that the self-managing communities of interest represent an institution which is both more efficent and more democratic than the existing systems of financing public expenditures.

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FINANSIRANJE JAVNIH SLUŽBI U JUGOSLAVIJI: MODEL LINDALOVE RAVNOTEŽE ZA EKONOMIJU RADNIČKOG SAMOUPRAVLJANJA

Dubravko MIHALJEK

Rezime

U ovom eseju smo predstavili model finansiranja javnih službi u Jugoslaviji. Model se sastoji od jednog reprezentativnog domaćinstva, preduzeća koje proizvodi potrošačka dobra, preduzeća koje proizvodi kolektivna dobra i institucije koja se naziva samoupravna interesna zajednica (SIZ), u okviru koje su korisnici i pružaoci javnih usluga pregovarali o proizvodnji i cenama kolektivnih dobara. Specificirana je Lindalova ravnoteža za ovu samoupravnu ekonomiju i tri alternativna vida finansiranja javnih izdataka upoređena su sa ovom ravnotežom. Vid finansiranja preko SIZ-a pokazuje se kao Pareto superioran u odnosu na sva tri vida finansiranja putem poreza. Ovo u prvom redu proizlazi iz odsustva izvitoperujućeg uticaja Lindalovih cena na izbor agenata, a što je samo posledica specifične prirode procesa pregovaranja u SIZ-u, koji je postavljen tako da bi osigurao postizanje Lindalove ravnoteže.

Ova analiza bi se mogla produžiti u nekoliko pravaca. Jedan od njih bio bi da se obogati struktura modela. Npr. mogla bi se postaviti eksplicitna scena pregovaranja u SIZ-u, a problemi optimizacije mogli bi se tako generalizovati da se izbegnu neke specifične posledice Kob-Daglas funkcije proizvodnje i korisnosti sa kojima se susrećemo u ovom eseju. Zatim, mogli bismo da posmatramo model u dinamičkom kontekstu, unutar koga bi zavisnost cena kolektivnih dobara od lagiranih varijabli uticala na aktuelne odluke agenata i stoga i na proces pregovaranja u SIZ-u. Na kraju, mogli bismo da izučavamo ovaj model u stohastičkom kontekstu, naročito što se tiče određivanja cena i količine proizvedenih kolektivnih dobara. Nadamo se da bi ovi novi pravci analize utemeljili mnoge zaključke do kojih se do sada došlo i potvrdili naše uverenje da samoupravna interesna zajednica predstavlja instituciju koja je ujedno i efikasnija i demokratičnija od ostalih sistema finansiranja javnih rashoda.