ted market, non-equalized economic conditions, escalating collective and general social consumption, stagnant rate of productivity and retarding rate of economic growth. The acts such as: the verifying and reprogramming of the social expenditures; discretional taxation of collective and general social consumption above the established social limits and material possibilities; accelerated depreciation; differential and, possibly, progressive taxation; the cutting down of excessive economic and social transfers; the institution of fiscal conjuncture duties; the destined use of the surpluses in socio-political communities and self-managing communities of interest for real supply stimulation; the discriminating fiscal grasping at the social income of economic sectors; the additional taxation of income resulting from exceptionally favourable circumstances (surtax); the taxation of investment exceeding social development plans; selective fiscal benefits and bonuses; long-term programmes for supporting export-orient sectors and developing priorities; the reduction of employment in socio-political communities, self-managing communities of interest and work communities; the productive allocation of internal and external debts, loans and credits; the compatible relation between the growth of fiscal burden and the growth of fiscal capacity, the correlative relation between the growth of personal incomes and the growth of social labour productivity, the reduction of the monetary growth rate; the deficit financing of invest- and accumulation; the elimination of the deficit financing of investment, as well as of collective and general social consumption, etc., have remained beyond the reach of the institutional solutions and in the background of the stop-go economic (and financial) strategy.

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CHARACTERIZATION OF COMPLETE EFFICIENCY IN A SPECIAL PROBLEM OF MULTICRITERION HYPERBOLIC PROGRAMMING

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1. INTRODUCTION

So far we have not got the characterization of complete efficiency in the general problem of multicriterion hyperbolic programming, that is, the necessary and sufficient conditions, under which the set of feasible solutions to the problem is equal to its efficient set, are not known. In this paper we give the characterization of complete efficiency in a special problem that appears in economic applications.

It is a question of the vector maximization problem:

$$max\ \{z = (z_1(x),\ z_2(x)\ ,z_3(x));\ x \in S\}. \tag{1}$$

$$z_k(x) = \frac{c^T x}{d^T{}_k x},\ (k = 1,\ 2,\ 3) \tag{2}$$

$$S = \{x \in R^n;\ Ax \leq b,\ x \geq 0\}, \tag{3}$$

where c^Tx is a value of production, while d_1 is vector of standardised, labour input, that is, of number of norm hours of living labour, d_2 is vector of unit production costs, and d_3 is vector of assets employed per unit of product. It is assumed that $x = 0 \notin S$. In this way not only $d^T{}_k x > 0$ holds, but $c^Tx > 0$ as well. Thus, the fficient solution to the problem (1)—(3) is such that it is not dominated by any other solution $x \in S$ according to the criteria of productivity, economy and profitability. (See ref. [2] page 90, and [3]).

The problem (1)—(3) can be linearized in the following way. First it is transformed into the minimum problem:

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$$\min \left\{ \frac{d^T_1 x}{c^T x}, \frac{d^T_2 x}{c^T x}, \frac{d^T_3 x}{c^T x}; \; x \in S \right\}. \tag{4}$$

If we assume that the set of feasible solutions S is bounded, Charnes-Cooper's method, known from one-criterion hyperbolic programming, can be applied to the problem (4). Thus, we get the following problem with three linear objective functions:

$$\min \{ d_1^T y, \; d_2^T y, \; d_3^T y \}$$
$$A y - b t \leq 0 \tag{5}$$
$$c^T y = 1$$
$$y \geq 0, \; t > 0$$

It is interesting that it cannot be found out whether the whole set of feasible solutions to the problem (5) is efficient, applying the characterization of complete efficiency in the multicriterion linear problem by M. Benveniste (1977). Namely, the set of feasible solutions to the problem (5) does not have an interior point, the existence of which is necessary to the above-mentioned characterization. (See M. Benveniste [1], theorem 3)[1] Thus, the inconvenience of Charnes-Cooper's linearization is in the fact that the original set of feasible solutions S is mapped into the set without an interior point (in the space R^{n+1}). Namely, all the points of the transformed set S lie on the hyperplane $c^T y = 1$.

2. CHARACTERIZATION OF COMPLETE EFFICIENCY

Now we shall see the conditions under which the set of feasible solutions M to the problem (5) is equal to the set E of efficient solutions to this problem.

THEOREM: Every feasible solution to the problem (5) is efficient, if, and only if, there is no vector $s \in R^n$ such that

$$d_k^T s \leq 0, \quad (k = 1, 2, 3) \tag{6}$$
$$c^T s = 0,$$

with strict inequality holding for at least one k.

[1] From the nature of the problem it follows that all variable coefficients in the objective functions are positive, that is, $d_k > 0$ for each k. Hence the vector $s \in R^n$ can be always found so that $Ds \leq 0$, where d^T_k is row k of matrix D.

Proof. The condition is sufficient, that is if there is no vector s so that (6) holds, then the set of feasible solutions M is efficient. Suppose on the contrary that the whole set M is not efficient. Then there exist points y and y_1 in M such that $d^T_k y \leq d^T_k y_1$, $(k = 1, 2, 3)$ $c^T y = 1$ and $c^T y_1 = 1$. But then, there is $s = y - y_1$, so that $d^T_k s \leq 0$ and $c^T s = 0$, which contradicts the assumption.

The condition is necessary, that is if $M = E$, then there is no $s \in R^n$ so that (6) holds. Suppose, on the contrary, that there exists vector s such that $d^T_k s \leq 0$, $(k = 1, 2, 3)$ and $c^T s = 0$. But then, from any interior point $y_1 \in M$, interior with respect to the hyperplane $c^T y = 1$, going in the direction s one can remain in the set M, that is, one can find sufficiently small scalar $\varepsilon > 0$ such that $y = y_1 + \varepsilon s \in M$. Hence $d^T_k y = d^T_k y_1 + \varepsilon d^T_k s$ and $d^T_k y \leq d^T_k y_1$, for every k, which contradicts the asumption that $M = E$. This completes the proof of the theorem.

COROLLARY. $M = S$ if, and only if, there is no solution to the system:

$$d^T_k s + v_k = 0, \quad (k = 1, 2, 3), \; e^T v \geqq 1, \; v \geqq 0, \tag{7}$$
$$c^T s = 0, \; s \in R^n, \; v \in R^3,$$

where e is vector of units.

In order to find out whether the system (7) has the solution, the well-known first phase of simplex algorithm is applied to this problem:

$$\min (w_1 + w_2)$$
$$v + Ds = 0, \; e^T v - u + w_1 = 1, \; c^T s + w_2 = 0 \tag{8}$$
$$v \geqq 0; \quad u, w_1, w_2 \geqq 0,$$

where $D = [d^T_k]$, $(k = 1, 2, 3)$, while u, w_1 and w_2 are variables, that is, $u, w_1, w_2 \in R^1$.

3. ILLUSTRATIVE EXAMPLE

Let us now illustrate the theorem on the following numerical example.

The following functions should be maximized

$$z_1 = \frac{3x_1 + 4x_2}{x_1 + 2x_2}, \quad z_2 = \frac{3x_1 + 4x_2}{5x_1 + 3x_2}, \quad z_3 = \frac{3x_1 + 4x_2}{8x_1 + 11x_2}$$

under constraints

$$3x_1 + 2x_2 \geq 18, \quad x_1 + 3x_2 \geq 13, \quad x_1 + 4x_2 \leq 26, \quad 3x_1 + x_2 \leq 23.$$

It is evident that the system

$$s_1 + 2s_2 \leq 0$$
$$5s_1 + 3s_2 \leq 0$$
$$8s_1 + 11s_2 \leq 0$$
$$3s_1 + 4s_2 = 0$$

with the sign $<$ in at least one inequation has no solution. Namely, the straight line $3s_1 + 4s_2 = 0$ does not pass through the cone determined by the system of inequations, in fact, by the first and second inequation. According to the theorem every feasible solution, that is, every point of the quadrilateral with the vertices in the points A (2, 6), B (4, 3), C (7, 2) and D (6, 5) is efficient solution. Our characterization of complete efficiency does not depend on the set of feasible solutions but on the objective functions. If the common numerator had been, for example, the function $2x_1 + 5x_2$ instead of $3x_1 + 4x_2$, only the extreme point A (2, 6) would have been efficient. Then $x_1 = 2$, $x_2 = 6$ would have been the perfect solution to the problem.

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MICRO PAYMENTS FLOWS AS A DATA SOURCE FOR THE NATIONAL ACCOUNTS[1]

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Yugoslavia's particular socio-economic setting offers possibilities to the compilation of national accounts and the use of computers which differ from the prevailing practices in Europe. I shall not refer here to the established official practice used in the compilation of Yugoslav accounts, which, although it offers many interesting aspects, particularly in the field of regional accounts, does not differ in principle from the traditional data sources. The aggregates which are usually used in the compilation of national accounts are provided by the statistical services, mainly by the aggregation of data from the micro accounts of firms and other transactors in the economy. Computers are widely used for this aggregation, but not for the compilation of the national accounts proper. Partly computerised are only the input-output tables.

Instead of the established practice, I'd like to report on some experimental work which tries to explore the particular advantages offered by the Yugoslav socio-economic system. This work consists of exploiting a new kind of micro data; the data on the flows of payment, which offer some particular possibilities to the national accountant and might therefore also be of more general theoretical interest. I first exposed the idea on which this work is based in the discussion at the 17th IARIW conference and later in a Yugoslav article.[2] Meanwhile, and independently, it was also adopted as one of the directions in the development of the official Yugoslav statistics.[3] The Federal Statistical Office does not follow these ideas yet by practical implementation on the national level, so I shall report on some empirical work being done along these lines in the region of Slovenia in the

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[1] Reported at the 18th general conference of the International association for research in income and wealth IARIW, Luxemburg, August 1983.

[2] Ivo Lavrač: *Nekatera možna izhodišča dela na sistemu bilanc za republiko.* IB revija za planiranje, št. 12, december 1981.

[3] Savezni zavod za statistiku: *Program razvoja statističkog sistema — zajedničke osnove.* Beograd 1982, str. 39.