matne stope na kredite i kamatne stope na štedne uloge što manje razlikuju. U tom slučaju, preduzeća bi se orijentisala u potpunosti na kreditno finansiranje investicija, dok bi radnići štedeli ulaganjem na štedne račune. Ukupan obim štednje i investicija ne bi bio niži od onog kad postoji lično vlasništvo nad sredstvima za proizvodnju u samoupravnom preduzeću.

Na kraju treba primetiti da, iako su u perfektnoj konkurenciji kredit i samofinansiranje podjednako dobri načini za finansiranje investicija (jer do samofinansiranja neće doći dokle god postoji neki projekat sa višom stopom prinosa od one koja je ostvariva unutar datog preduzeća), u stvarnosti je kredit preferabilan. Razlog za to je što se radnici u datom preduzeću mogu, usled subjektivnih faktora, pre odlučiti za ulaganje u soptsveno preduzeće, čak i kad je stopa prinosa niža od one koja je ostvariva van preduzeća. Naravno, takva situacija je, sa društvenog stanovišta, suboptimalna.

THE SELECTIONS OF ELEMENTS FROM A GIVEN SET RELATIVE TO ONE CRITERION

*Branislav IVANOVIĆ**

1. INTRODUCTION

At present we are increasingly encountering problems concerning the identification of one subset from a given set of elements that would, as a separate entity relative to some criterion, represent an extreme group of that set.

An example of such a problem would be, in the first place, the selection of the best or weakest elements relative to one or more variables, or relative to one common or synthetic criterion.

Problems of this sort are found in the everyday practice of numerous social, scientific and economic activities. For example, we could say that this problem area is the foundation of the policy for personnel promotion in administration, in the economy, in cultural fields, in the military, etc. The same is also true regarding the selection of candidates for job posts, school entrance exams, the organization of various representational groups, drawing up guest lists for receptions or meetings, approving individual items in investment or budget plans and, in general, when giving priority to individuals or categories.

It is obvious that problem-solving will be rendered more difficult if we have to deal with one multidimensional or synthetic criterion because the question is then raised of selecting the variable as well as an adequate synthetic criterion. Much discussion has already been devoted to these issues[1,2] so there is no need to dwell further on them here.

This group of problems also includes the very topical issue of nationalization of banks and industrial enterprise groups in France. In addition to 139 foreign banks, there are presently 111 national

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[1] B. Ivanović, "Problème de l'identification des pays les moins avances parmi les pays en voie de développement", Conférence des Nations Unies sur le commerce et le développement (CNUCED), Genève, 1970.

[2] B. Ivanović, "Comment établir une liste optimale des indicaturs de développement", Revue de Statistique Appliquée, No. 2, Paris, 1974.

banks in France. The program of the French Government envisaged the nationalization of 36 national banks while considering the remaining 75 to have a "popular" character. The reaction was violent. According to which objective criterion was a bank declared to be "nationalizable", and why 36 and not, for example, 35 or 37 banks? Fifty thousand stockholders of Crédit Commercial de France (which was up for nationalization) stated in their protest that "the authors of the program have never been able to, nor will they ever be able to, justify the discrimination between "nationalizable" and "popular" banks." The stockholders therefore hold that if it is really necessary, then all banks should be nationalized, and if not then no bank should be nationalized. It is obvious that neither of these extreme resolutions suits the Socialist Party in government because the first is in the "final" program of the French Communist Party and the second is in the context of the capitalist ideology prevailing in the right-wing parties. The program of the Socialist Party can certainly be criticized for not elaborating objective criteria for quantitatively determining the degree of the society's need for the nationalization of a single bank, and then criteria for separating an objective, clearly-discriminated group of banks that will be nationalized and will represent a separate entity. Otherwise, the impression remains of irresponsible arbitrariness on the side of the Socialist Party in adopting concrete decisions with regard to nationalization. The same criticism, although with perhaps less severity, could also be addressed to the "nationalizable" industrial enterprise groups.

In determining the group of weakest elements in a given set, the problem can first be raised of identifying the most poorly-developed regions in a country — which are increasingly being treated in the framework of the socio-economic advancement of that country — in order to extend special aid to the whole country for the purpose of accelerating the development of the weakest regions and thus closing the gap between the richest and poorest regional units.

Yugoslavia (1956) and several other countries have acquired a certain amount of experience in ranking and identifying their weakest regions. In light of the highly sensitive nature of the problem being treated, because it is always a matter of substantial financial and social aid to regions in the group of the weakest, this experience has shown that every solution must be anchored on irreproachable scientific-objective argumentation and must be free of any arbitrariness. Otherwise, controversies are inevitable and the final result will have an impact completely different from the one desired: instead of rapprochement, the regions will grow apart with a swelling feeling of bitterness and injustice.

An analogous problem is the one of elimination in personnel policy, especially in cases where each candidate is tested by measuring a series of traits, and if a primitive method of adding up arbitrarily-defined points is not employed. As the case in point here concerns the career and frequently the very survival of the individual, it is unnecessary to stress the amount of delicacy, caution and objectivity required in attempting to resolve such problmes of elimination.

Problems of selection also occur on the international plane. We could cite here one that is linked to drawing up a list of the poorest among the developing countries (a project undertaken by the UN Committee for Development Planning) as well as the problem of identifying the group of developing countries most sorely affected by the oil crisis (the UN Commission for Trade and Development).

Selecting the group of best elements, relative to some criterion, will be greatly facilitated if that criterion can be quantified and an ordered classification of all the elements in the observed set is established. It is evident then that the first elements of that order will belong to the group of best elements, and that the latter ones will belong to the group of weakest elements. The problem, however, is still unresolved because the task remains to draw the borders in that order which would separate the group of best, or the group of weakest, elements.

Generally, the problem is indefinite because if there are no other additional constraints, such as a pre-fixed number of elements in the selective group or a pre-fixed borderline value of the criterion, the number of possible solutions will be equal to the number of elements in the observed set.

The goal of this study is to present several procedures enabling either the complete solution to the selection problem raised or a reduced degree of its indefiniteness. In the second case, the significance of such results lies in the following: if the observed set contains twenty elements and if, for example we have reduced the number of alternative solutions from a total of twenty to only four, then it is obviously much easier to coordinate the opinions of all the interested parties and to accept one mutual solution of the four possible, clearly-separated ones.

Therefore, for the obtained order according to the given criterion, and by using a matrix of similarity relative to that criterion, we can acquire a Sorensen dendogram defining a hierarchical classification of the observed set. By using that hierarchy we can form a reduced ordered set of parts whose core will be the last element in that order. The employment of this procedure can substantially reduce the number of alternative solutions, i.e., it can decrease the degree of indefiniteness of the problem being examined.

In an analogous way we can also solve problems where the order of the elements of the basic (observed) set is determined via one criterion and the dendrogram via another.

It frequently occurs in practice that all the interested parties feel that certain elements should be part of the group of the weakest ones. Then the group of the weakest will comprise the most restrictive class defined by these elements and by the weakest element in the order relative to Sorensen's dendogram.

If the elements of the basic set of the statistical mass with the given orders are according to the observed criterion, and if we wish to use all available information, the problem will be solved in a somewhat different fashion. More precisely, we will determine the ordered classification of populations via their arithmetical means, and the matrix of separability coefficients will correspond to the matrix of similarity. The obtained dendrogram will enable us to arrive at a sequence of alternative solutions for determining the group of weakest populations while giving consideration to the separateness or blending of the masses existing between them. The number of these alternative solutions is lower than the number of populations, and this decreases the degree of arbitrariness in separating the group of the weakest; also, every broader solution contains all the populations of the proceding solution. The given severity criterion for degree of separability can influence our decision to opt for one of them.

Also observed is the case where the order of elements of the basic set is formed via one criterion, and where an examination must be made of a set of alternative solutions for dividing that basic set into the group of weakest elements and the group of its remaining elements — via one multidimensional feature. This problem is also solved by applying coefficients of separability.

The method that we have proposed here provides an effective instrument for checking the revision of the group of weakest elements to show whether or not a really objective improvement has been attained.

By using this method we can define the best separated group of weakest elements. More precisely, if the number of elements in the group of the weakest is not essential, then the group of the weakest will be that to which the coefficients of separability correspond *maximum maximorum.*

The most clearly-expressed problem in determining the group of weakest elements in the basic set, according to one observed set of variables, occurs in the case where all these variables are used to construct the criterion by which the ordered classification of all the elements of the basic set is obtained. Once the order is established, the determination of the group of weakest elements will be made via the Sorensen dendogram — which is a more thorough procedure than the one with the coefficient of separability.

Finally, mention is made of the rather sophistical combined method of F.I.[1] in indentifying the weakest elements of the basic set.

In closing', it is noteworthy that we also encounter this problem area in the Yugoslav self-management system of compacts. More precisely, if a social compact is made concerning some economic issue, then it is useful to give a corresponding scientific-logical structure to that policy solution in order to avert all contradictions and any possible undesirable consequences.

2. DETERMINING THE GROUP OF WEAKEST ELEMENTS IN AN OBSERVED SET RELATIVE TO ONE CHARACTERISTIC

Let us observe set S with N elements, for which we are measuring characteristic X. If we arrange the obtained numerical values

by magnitude, we denote by e_i that element whose value $X = x_i$ has i *th* rank in that order. The monotonic increasing sequence

$$K_S = < x_1, x_2, \ldots . x_N >$$

represents the ordered classification of set S relative to characteristic X.

Now let us denote

$$A_{j1} = \{x_1, x_2, \ldots, x_j\}\ , \qquad j \in \{1, 2, \ldots, N\}\ ,$$

whereby

$$A_{11} = \{x_1\}\ , \qquad A_{N1} = S\ , \qquad A_{j1} \subseteq A_{j+1,1}\ .$$

As

$$\bigcap_{j=1}^{N} A_{j1} = \{x_1\}\ ,$$

this will say for element x_1, i.e., for the weakest element in set S relative to X, that it represents the core of the arranged set of S parts.

$$K_A = < A_{11}, A_{21}, \ldots, A_{N1} > .$$

The determination of the group of weakest elements of S, relative to X, is indefinite because each of the subsets A_{j1} ($j \in \{1, \ldots, N\}$) can represent one such group. More precisely, there are as many solutions (N) as there are elements.

We will therefore try to reduce the number of possible logical solutions.

If we understand the absolute difefrence $d_{ij} = |x_i - x_j|$ to be the degree of similarity between e_i and e_j relative to X, then we can form the dendrogram of set S via the matrix of similarity $D = [d_{ij}]$ and by employing the method of complete linkage. As the order of elements is already fixed with K_S, the first neighboring diagonal with a principal diagonal of D will be used to form that dendrogram.

Therefore, Sorensen's dendrogram, which defines a hierarchical classification of set S, corresponds to every K_S sequence.

Let $x_1 \neq x_2$ and we denote $\{x_1\} = B_{11}$. Further, by d_1^* we denote the level at which the element x_1 links the neighboring group and forms group B_{21} and, in general, d_j^* denotes the level at which group $B_{j-1,1}$ links with the neighboring group and forms group B_{j1}. Let there be a total of n such groups. It is evident that

$$B_{11} = \{x_1\}\ , \qquad B_{n1} \div S\ , \qquad n \leqslant N \quad i \quad B_{j1} \subseteq B_{j+1,1}\ .$$

As

$$\bigcap_{j=1}^{n} B_{j1} = \{x_1\}\ ,$$

x_l is the core of the arranged set of parts

$$K_B = \langle B_{1l}, B_{2l}, \ldots, B_{nl} \rangle \ .$$

Every B_{jl} comprises one entire class of the weakest elements of S, both in light of the similarity existing among them and of the criterion for linking d_j^*.

Thanks to this method, the number of alternative solutions is reduced from N to n.

The problem will be completely resolved if we add one of the following conditions:

1° that the strictness of the criterion for linkage must not be lower than a given border,

2° that the number of elements in the weakest group is not higher than a pre-fixed number,

3° that the value of X elements in the weakest group is not higher than a pre-fixed number.

In other words, these conditions are reduced to

1° $d^*_m \leqslant d$

2° $|B_{ml}| \leqslant k$

3° $\forall_j \, [j \in \{1, \ldots, m\} \; i \; e_j \in B_{ml} \Rightarrow x(e_j) \leqslant c]$.

where d, k and c are the constants given in advance.

As an example we take set S, whose dendrogram is shown in Fig. 1.

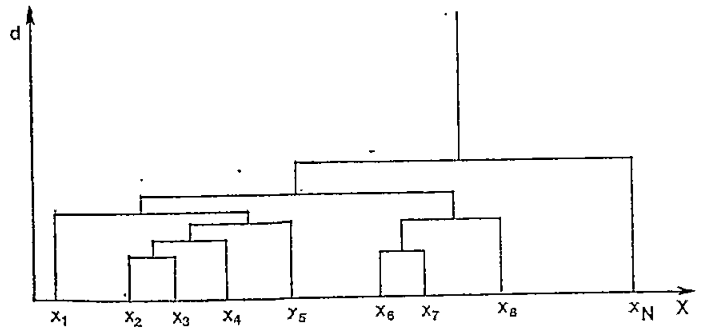

Figure 1

If, for some specific reasons, the number of elements of the weakest group should not be higher than eight, possible alternative solutions will be:

$B'_{1l} = \{e_1\}$ on the level $d_1^* = 0$,

$B'_{2l} = \{e_1, e_2, e_3, e_4, e_5\}$ on the level $d_2^* = 6$,

$B'_{3l} = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ on the level $d_3^* = 7$.

The number of alternative solutions is reduced from eight to three, so the conclusion follows that the new procedure for determining the group of weakest elements of S relative to X, although still with a certain amount of arbitrariness, is more precise than the arbitrary drawing of borders in the ordered classification K_S.

3. THE METHOD OF THE MOST RESTRICTIVE CLASS

Let $K_S(Y)$ be the ordered classification of set S of N elements, obtained via criterion Y. Determine the group of weakest elements according to Y relative to criterion X.

For example, S can be the set of regions of one country, Y the social income *per capita*, and X the rate of growth of the industrial production of the regions. Maintaining the order $K_S(Y)$, determine the group of the weakest regions of the observed country via criterion X.

If $s_X(e_i, e_j)$ is the selected measure of similarity, then, maintaining the order $K_S(Y)$, the dendrogram of the hierarchical classification of set S (Fig. 2) can be determined via the corresponding matrix of similarity $\varphi_X = [s_X(e_i, e_j)]$

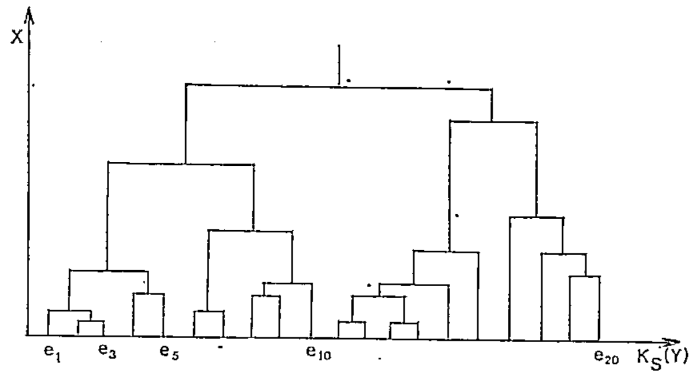

Figure 2

In the example whose dendrogram is given in Fig. 2, we will have the following alternative solutions for the group of weakest elements of S:

$B'_{1l} = \{e_1\}$,

$B'_{2l} = \{e_1, e_2, e_3\}$,

$B'_{3l} = \{e_1, e_2, e_3, e_4, e_5\}$,

$$B'_{41} = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$$

$$B'_{51} = \{e_1, e_2, \ldots, e_{20}\} = S.$$

It often occurs in practice that all the interested parties consider that element $e_k, e_r, \ldots, e_m$ should join the group of the weakest. Then that group will be comprised of the most restrictive class[3] of elements (e_1, e_k) or the elements $(e_1, e_k, e_r, \ldots, e_m)$ of the hierarchical classification of set S — given by the dendrogram in Fig. 2. Therefore, the solution in the first case will be $B^*(e_1, e_k)$, and in the second $B^*(e_1, e_k, e_r, \ldots, e_m)$.

In our example, if we consider that element e_2 should join the group of the weakest, then that group will be determined by the most restrictive class $B^*(e_1, e_2) = B'_{21}$.

Similarly, if we consider that the group of the weakest should contain element e_6 in addition to e_2, the solution will be

$$B^*(e_1, e_2, e_6) = B'_{41}.$$

In other words, if there is agreement that element e_2 should join the group of the weakest in addition to the weakest element e_1, then element e_3 should also join this group due to mutual similarity.

By the same token, if agreement is reached that elements e_2 and e_6 should join the group of the weakest with e_1, then, due to mutual linkage, the elements $e_3, e_4, e_5, e_7, e_8, e_9$ and e_{10} must also join that group.

Here we also encounter the Yugoslav self-management system of compacts. More precisely, if a social compact is made concerning some economic issue, then it is necessary to give a corresponding scientific-logical structure to that policy solution in order to avert any contradictions and possible unjust consequences. For example, if agreement has been reached that e_1, e_2 and e_6 are taken as the weakest elements, the group of weakest elements cannot be formed of only those three elements because it would be contradictory to overlook e_3, e_4 and e_5 and unjust to ignore e_7, e_8, e_9 and e_{10} which are inseparably linked to e_6.

4. DETERMINATION OF THE GROUP WEAKEST POPULATIONS IN ONE SET RELATIVE TO ONE VARIABLE

Let us now observe the case where the elements of set S are populations and we are measuring variable X of these elements. Then, an order with the arithmetical mean $\bar{x}_i$ and the law of probability $f_i(x)$ will correspond to every element e_i. By arranging the arithmetical means by magnitude, we will again obtain a monotonic increasing sequence

[3] B. Ivanović, "Groupement des pays par rapport a leurs profils socio-économiques", UNCTAD, Geneva 1971.

$$K_S = \langle \bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N \rangle,$$

which represents the ordered classification of set S relative to X.

How do we now separate the poorest-developed population of set S relative to X?

The difference between neighboring arithmetical means is no longer important, but rather the knowledge of whether or not there is a mixture of their orders among neighboring populations. More precisely, despite a small interval between the arithmetical means of two neighboring populations, they will be considered remote from each other if there is no mixture between the masses of their orders.

For example, in Fig. 3 we see that populations e_1 and e_2 are completely separated and so are less close than populations e_2 and e_3.

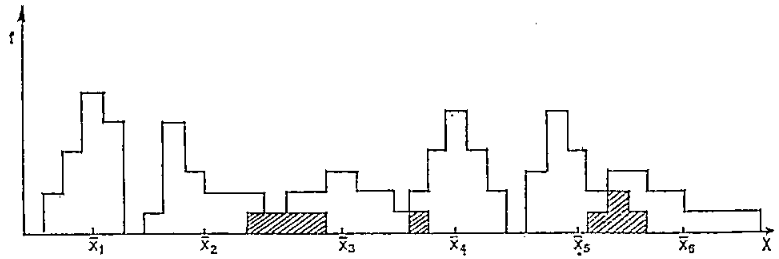

Figure 3

Therefore, instead of distance d_{ij}, we will take the measure of separability[4] as the measure of similarity between the two populations of order K_S.

The measure of separability reacts to every mixture. It is sufficient for only one element of one population to wander into the region of another population to cause a reduction in its value.

Let us first assume that the population orders are continuous and that their laws of probability are known to us. Let us denote the law of probability of population e_i by $f_i(x)$, and its arithmetical mean by $\bar{x}_i$.

We will define the measure of separability between populations e_1 and e_2, with the respective laws of probability $f_1(x)$ and $f_2(x)$, by

$$(4.1)\quad \tau_{12} = \frac{E(Y) - E(X)}{E(Y - X)} = \frac{\bar{y} - \bar{x}}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |y - x|\, f(x, y)\, dx\, dy},$$

[4] B. Ivanović, »Izbor obeležja prema njihovom stepenu separabilnosti u odnosu na posmatrane statističke skupove« (The Selection of Variables According to their Degree of Separability Relative to Observed Statistical Sets), The VIIIth Annual Meeting of the Yugoslav Statistical Society, Zagreb, 1967.

whereby $f(x, y)$ is the two-dimensional law of probability whose marginal laws are $f_1(x)$ and $f_2(y)$, and $\bar{x} \leqslant \bar{y}$.

If both distributions are completely separated:

$$|x - y| = y - x \Rightarrow \tau_{12} = 1.$$

If both distributions are completely mixed: $\bar{x} = \bar{y} \Rightarrow \tau_{12} = 0$.

In the general case, $0 \leqslant \tau_{12} \leqslant 1$.

$|y - x| = y - x$ $\qquad$ $|y - x| < y - x$

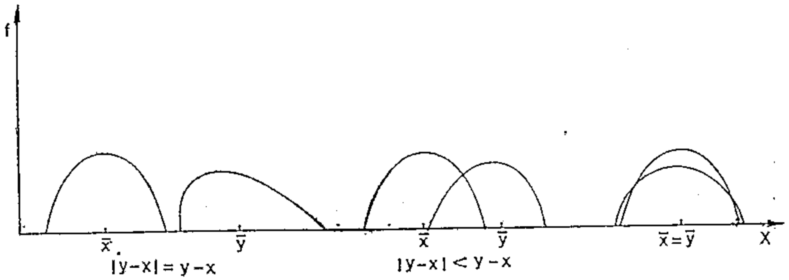

Figure 4

Let us now assume that the population distributions are not continuous and that their elements are grouped according to group intervals.

The measure of separability between populations e_1 and e_2, with respective non-continuous laws of probability $\{<x_i; f_i^{(1)}>\}$ and $\{<y_j; f_j^{(2)}>\}$, will be

$$(4.2) \quad \tau_{12} = \frac{\bar{y} - \bar{x}}{\sum_i \sum_j |y_j - x_i| \cdot f_{ij}},$$

where f_{ij} is the relative frequency of the two-dimensional distribution and $\bar{x} \leqslant \bar{y}$.

If both distributions are completely separated:

$$[|y_j - x_i| = y_j - x_j] \Rightarrow \tau_{12} = 1.$$

If both distributions are completely mixed:

$$\bar{x} = \bar{y} \Rightarrow \tau_{12} = 0.$$

And here, in the general case, $0 \leqslant \tau_{12} \leqslant 1$.

The procedure for determining the weakest group of the populations of set S will be carried out by forming the dendrogram via the ordered classification $K_S = <\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N>$ and the matrix of separability $T = [\tau_{ij}]$. The neighboring diagonal of the principal diagonal of T gives the coefficients of separability between the neighboring populations ranked according to K_S.

The obtained dendrogram enables us to arrive at a series of alternative solutions for determining the group of weakest populations, giving consideration to the separation existing between them. The number of these alternative solutions is lower than the number of populations of S, whereby the degree of arbitrariness in separating the group of the weakest is reduced and every broader solution contains all the populations of the preceding solution. The given criterion of severity of the degree of separability will influence us in opting for one of them.

If an agreement is made to include another specific number of populations in addition to e_1, the group of the weakest will be determined by the method of the most restrictive class.

As is evident, the problem of determining the group of weakest populations in set S relative to variable X is methodologically resolved in the same way as in the preceding case, except that now — knowing the distributions of elements in set S — we have additional information that we use for determining more precise relations among these elements and, by extension, for obtaining more precise results.

Regarding the practical application of this method, if the value of $x_i^{(1)}$, or $y_j^{(2)}$, is a high number, the calculation of coefficient τ_{12} becomes a painstaking task. Then the whole procedure can be simplified by giving the following statistical form to the coefficient of separability

$$(4.3) \quad \tau_{12} = \frac{\bar{y} - \bar{x}}{\sigma_y - \sigma_x}$$

This so-called dispersion measure of separability between two populations[5] varies in the interval $[0; +\infty]$ and achieves a value of zero if the masses of both distributions are completely mixed $(\bar{y} = \bar{x})$. It is said for $\tau_{12} \leqslant 1$ that the populations e_1 and e_2 are poorly separated, and that for $\tau_{12} > 1$ that they are clearly separated.

5. DETERMINATION OF THE GROUP OF WEAKEST ELEMENTS OF ONE SET RELATIVE TO MULTIPLE VARIABLES

Let us assume that on the basis of criterion Y we have determined the ordered classification of N elements of set S, and that two alternative solutions H_1 and H_2 already exist for the division of that

[5] B. Ivanović, »Teorija klasifikacije« (The Theory of Classification), Institut za ekonomiku industrije (The Institute for Industrial Economics), Belgrade, 1977.

set into the group of the weakest elements and into the group of its remaining elements.

Let

$$A_{mI} = \{e_1, e_2, \dots, e_m\},$$
$$A_{nI} = \{e_1, \dots, e_m, e_{m+1}, \dots, e_n\}, \quad m < n,$$
$$m, n \in \{1, 2, \dots, N\}$$

so that the alternative solutions

$$H_1 = \{A_{mI}, A'_{mI}\} \quad \text{and} \quad H_2 = \{A_{nI}, A'_{nI}\}.$$

Examine which division is better, in other words, which group is better separated from the other elements of set S.

Let us assume that the variables $X_1, X_2, \dots, X_k$ will be used for that examination, and that we dispose of their statistical data for all the elements of S.

We adopt that the H distribution is as good as the separation between the group of elements $A = \{e_1, e_2, \dots, e_m\}$ and $A' = S \setminus A$. If we use τ_p to denote the coefficient of separability between these groups relative to variable X_p, the coefficient of separability relative to all k variables will be

$$(5.1) \quad \tau(k) = (\prod_{p=1}^{k} \tau_p)^{1/k},$$

whereby

$$(5.2) \quad \tau_p = \frac{m(N-m)(x^{-p}{}_{A'} - x^{-p}{}_{A})}{\sum_{i=1}^{m} \sum_{j=m+1}^{N} |x^p{}_{A'i} - x^p{}_{Aj}|}$$

The value of coefficient $\tau(k)$ varies from 0 to 1. In the case of complete separation of accumulation of points of elements of groups A and A' in R^k, it will be $\tau(k) = 1$. In the case of the coincidence of their centers of gravitation in R^k or in one of its subspaces, it will be $\tau(k) = 0$.

If now $\tau_i(k)$ is the coefficient of separability for division H_i relative to the set of variables $X = \{x_1, \dots, x_k\}$, and if $\tau_i(k) > \tau_j(k)$, the division H_i is better than the division H_j.

We notice that for the determination of distribution K_S the criterion Y can be identical to one of the variables of set X. For example, in determining the group of the most poorly-developed regions of a single country, we can take the *per capita* social income for the criterion $Y = X_1$, and for X the set of k indicators of socio-economic developmental level. However, as then $\tau_1 = 1$, X can be reduced to the set

$$\{x_2, x_3, \dots, x_k\}.$$

By the same token, this method can be used for investigating two or more divisions of set S without any order of their elements. But this is no longer a matter of the group of weakest elements, rather only of which of the alternative groups is better separated from the group of the remaining elements of set S.

Example. — In 1972, UNDP charged its working group, with Professor Jan Tinbergen at the head, to determine the group of most poorly-developed coutries (A_1) from among the developing countries (S). After a long and very controversial discussion, this working group stayed with an information base of only three indicators of development:

x_1 : *Per capita* national income,
x_2 : Percentage of illiterates over the age of ten, and
x_3 : The share industrial production in the national income.

The fourth indicator, „The Per Capita Rate of Growth of the National Income", had the occasional role of a corrective factor.

Without going into the validity and justification of a primitive procedure, which Tinbergen's group used for the identification of the weakest countries, the following list of countries was obtained. It is supposed to represent the group of most poorly-developed countries.

A_1 = {Upper Volta, Burundi, Rwanda, Yemen. Chad, Mali, Ethiopia, Somalia, Malawi, Niger, Laos, Nepal, Dahomey, Afganistan, Tanzania, Gambia, Haiti, Botswana, Sudan, Guinea, Uganda, Lesotho and Togo}.

The UNDP sent this proposed list to ECOSOC for further deliberation. Thanks to its hierarchical authority, ECOSOC made certain changes in the list. More precisely, Gambia and Togo were removed from the list while Bhutan, Western Samoa, Sikkim and the Maldives were included.

Thus, list A_2 was obtained. This was accepted by the UN General Assembly with the proposal that intensive aid be extended to the countries on that list.

A_2 = {Upper Volta, Burundi, Rwanda, Yemen, Chad, Mali, Ethiopia, Somalia, Malawi, Niger, Laos, Nepal, Dahomey, Afghanistan, Tanzania, Haiti, Botswana, Sudan, Guinea, Uganda, Lesotho, Bhutan, Western Samoa, Sikkim and the Maldives}.

Taking UNCTAD's set of 11 indicators of socio-economic development, the coefficient of separability of division H_1, of the division between the countries on list A_1 and the other developing countries, amounts to $\tau_1(11) = 0.967$. On the other hand, the coefficient of separability between the countries on ECOSOC's list A_2 and the other developing countries amounts to $\tau_2(11) = 0.965$. This means that

ECOSOC's corrected group was actually more poorly separated than the group of countries proposed by the UNDP.

Therefore, regardless of how list A_1 was compiled, it is still better than ECOSOC's list A_2, and so the question is raised of what motivated ECOSOC to compile a list that was worse than Tinbergen's.

In any case, the method that we have proposed here offers an effective means for control when revising a group of weakest elements, for determining whether or not a revision has truly brought about an objective improvement.

Finally, we note that for the given order K_S, by using this method we can seek the best-separated group of weakest elements of set S, relative to criterion Y, and for the set of variables X — via coefficients of separability.

Let us dwell on the first and weakest elements of order K_S with the denotation $A_i = \{ e_1, e_2, \ldots, e_i \}$ and $A'_i = \{ e_{i+1}, e_{i+2}, \ldots, e_N \}$.

For each division $H_i = \{ A_i, A'_i \}$ we will obtain the corresponding value of the coefficient of separability τ_i (k), and $i \in \{ 1, 2, \ldots, N-1 \}$, calculated on the basis of k variables of set X. The diagram of the sequence of coefficients of separability obtained in this way will represent an interrupted line with a certain number of minimums and maximums. Let n be these maximums and let

$$\forall_j [\, j \in \{ 1, 2, \ldots, n \} \perp M_j \in \{ 1, 2, \ldots, N \} \Rightarrow \tau_{M_{j-1}} < \tau_{M_j} > \tau_{M_{j+1}} \,]$$

Between every two successive and different-value minimums of coefficients of separability τ_{m_s} and $\tau_{m_{s+1}}$ there is a maximum, i.e., there is a division along the corresponding sub-interval $[m_s, m_{s+1}]$ which best separates the group of weakest elements.

If the number of elements in the group of the weakest is not important, then the best-separated group of the weakest will be the one that *maximum maximorum* corresponds to the coefficient of separability.

Figure 5

In the example in Fig. 5 we see that for the observed set of 16 elements the group of the weakest can be formed in six possible ways:

Rank of size of group	Size of group	Coefficient of separability
0— 2	1	0.167
2— 4	3	0.375
4— 8	5	0.583
8—12	10	0.917
12—14	13	0.833
14—16	15	0.667

The weakest five and ten elements are most evidently separated, and the coefficient of separability *maximum maximorum* corresponds to the group of ten elements.

6. DETERMINATION OF THE GROUP OF WEAKEST ELEMENTS OF ONE SET RELATIVE TO ONE SYNTHETIC CHARACTERISTIC

Criterion Y can also be a synthetic variable derived via the variables of set X. If only some of the variables of X are used in the formation of that characteristic, i. e., if

$$Y = Y(X_1, \ldots, X_k) \; i \; \{ X_1, \ldots, X_k \} \subseteq X,$$

then the procedure for determining the group of the weakest remains identical to the procedure presented in Paragraph 5.

For example, in the latest investigations being carried out by the UN Secretariat, the goal is to identify that group of developing countries hit hardest by the oil crisis. In the framework of UNCTAD, over time the opinion has crystallized that it would be adequate to use the following variables in these investigations:

X_1 — *Per capita* national income,
X_2 — Foreign trade loss expressed in terms of trade,
X_3 — Rate of growth of import volume,
X_4 — Rate of growth of export volume,
X_5 — The ratio between debt repayment and value of exports, and

X_6 — The participation of industrial goods in total exports.

During consultation meetings at UNCTAD, I proposed that the following parameter be taken as the measure of the vulnerability of a country caused by the oil crisis:

$$Y = \left[\frac{Debt}{National\ income} \, x \, \frac{Import}{Export} \right]^{\tau_u/\tau_i} \qquad (6.1)$$

where τ_u is the rate of growth of import and τ_j is the rate of growth of export. The higher the value of Y, the more the observed country will be handicapped, and there will be no damage if Debt = 0.

We note that the information contained in criterion Y is a part of the total information offered by the set of variables X.

By using the above-mentioned six variables of X, we will determine, for the given order K_S of the developing countries relative to Y, the coefficient of separability τ_j (6), $j \in \{1, \ldots, N\}$ and thus determine the group of maximally handicapped countries. If the rank of size of group is not conditioned in advance, it will be determined by the *maximum maximorum* coefficient of separability; if the rank of size is fixed in advance, the group of most handicapped countries will be defined by the maximum coefficient of separability in the framework of the corresponding sub-interval $[m_s, m_{s+1}]$.

The most highly-expressed problem in determining the group of weakest elements in set S, on the basis of the observed set of variables X, occurs in the case where all these variables are used for the synthetic formation of criterion Y, i. e., where

$$Y = Y(X_1, \ldots, X_n) \quad \{X_1, \ldots, X_n\} = X.$$

Once the ordered classification of elements of S relative to Y is established, the determination of the weakest group can be made via the dendrogram, which is a more thorough procedure than the one with the coefficient of separability.

A case in point would be the identification of the most poorly-developed countries among the developing countries by applying the method of I-distance based on a set of indicaitors for socio-economic development $X = \{X_1, \ldots, X_n\}$. Here criterion Y is the degree of socio-economic development which is demonstrated in the form of the I-distance between the observed country and the fictitious most poorly-developed country, i. e., the fictitious country whose values of selected indicators correspond to the minimal value of X in set S,

$$Y = Y(X_1, \ldots, X_n) = \sum_{i=1}^{n} \frac{X_i - \bar{x}_i}{\sigma_i} \prod_{j=1}^{i-1} (1 - r_{ij \cdot 12 \ldots j-1}) \tag{6.2}$$

where $\bar{x}_i$ is the minimal value of the indicator X_i in set S, σ_i is the standard deviation of X_i, and $r_{ji \cdot 12 \ldots j-1}$ is the partial coefficient of correlation between X_j and X_i for the fixed values $X_1, X_2, \ldots, X_{j-1}$.

For the obtained order

$$K_S = \langle Y_1, Y_2, \ldots, Y_N \rangle,$$

where Y_r is the I-distance between country e_r and the fictitious weakest country e^-, it will not be possible to determine a direct dendrogram, i. e., via direct differences between the successive members of

that sequence. More precisely, these differences are not simultaneusly the corresponding I-distance because, as we see in Fig. 6, for each $r \in \{1, \ldots, N\}$

$$Y(e_r, e_{r-1}) \geq Y_r - Y_{r-1}.$$

This is why, in order to form the dendrogram, a calculation must first be made of the I-distances of the neighboring diagonal of the principal diagonal of the distance matrix.

With a dendrogram determined in this way, we will obtain a series of alternative solutions for establishing the group of most poorly-developed countries among the developing countries

$$K_B = \langle B'_{1l}, B'_{2l}, \ldots, B'_{lnl} \rangle,$$

where $m \leq N$. We will be able to pinpoint the very group of the poorest-developed if the borderline of strictness of the criterion for linkage is given, or if the rank of size of group of the weakest is given. Similarly, if agreement is reached that countries $e_k, e_r, \ldots, o_s$ should join the group of the weakest, then the group of the poorest-developed countries will be represented by the most restrictive class of the hierarchical classification of the obtained dendrogram which contains the elements $e_1, e_k, e_r, \ldots, e_s$.

Finally, if we are given no pre-conditions, the group of poorest-developed countries among the developing countries can be represented by that group B'_{r1} for which

$$\max_{1 < r < N-1} \{ |B'_{r+1,1}| - |B'_{r1}| \}, \tag{6.3}$$

because that class is most obviously separated on the dendrogram.

For example, on the Fig. 2 dendrogram, the best-separated group of the weakest would be represented by group B'_{41} because the difference $|B'_{r+1,1}| - |B'_{r1}|$ is the biggest for $r = 4$.

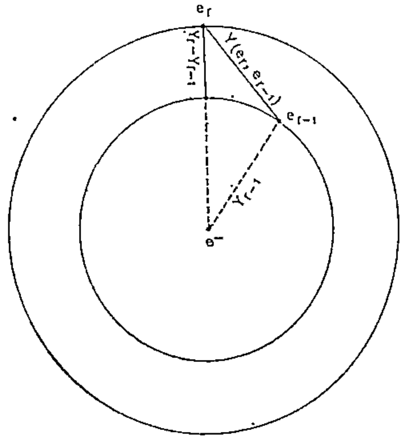

Figure 6

7. THE APPLICATION OF THE COMBINED METHOD OF F. I. FOR IDENTIFYING THE GROUP OF WEAKEST ELEMENTS IN AN OBSERVED SET[1]

Again we assume that on the basis of criterion Y we have determined the ordered classification K_S of the elements in set S and that we wish to separate the group of weakest elements using the set of variables X. In the example of identifying the group of most poorly-developed countries, Y can be the I-distance, calculated on the basis of variables $\{X_1, \ldots, X_k\}$.

We denote by C_1 the set of those elements for which all the interested parties agree that they should join the group of the weakest; we denote by C_2 the set of those elements for which it has been agreed by all that they should not be included in the group of the weakest. It is obvious that

$$C_1 \cap C_2 = \varnothing \qquad C_1 \cup C_2 \subseteq S.$$

Sets C_1 and C_2 represent two accumulations of points in space R^k. We denote by $\overline{X}'$ and $\overline{X}''$ the centers of gravitation of sets C_1 and C_2 in R^k, and by M the weighted arithmetical mean of these centers. Let W be the dispension matrix of X and $d_j = x'_j - x''_j$.

Fisher's hyper-plane of discrimination

$$\sum_{i=1}^{k} \sum_{j=1}^{k} w^{ij} d_j (X_i - M_i) = 0$$

separates in the best possible way the accumulation of points C_1 and C_2 in space R^k.

Now we include the other points of set S. Fisher's hyper-plane divides space R^k into two such regions that we accept that the investigated point belongs to the group of the weakest if it is found in the region R^k_1, i. e., in the region on which $\overline{X}'$ is found.

We define the group of weakest elements of set S in the following way:

The group of the weakest will be formed by those elements of S whose points are found in region R^k_1 and whose rank in K_S is below the rank of all the elements whose points are found in R^k_2.

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SELEKCIJA ELEMENATA IZ DATOG SKUPA U ODNOSU NA JEDAN KRITERIJUM

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Rezime

Predmet ovog rada je rešavanje problema identifikacije jednog podskupa iz datog skupa elemenata koji bi, kao zasebna celina u odnosu na neki kriterijum, predstavljao jednu ekstremnu grupu toga skupa.

Takav problem bi, pre svega, bio selekcija najboljih ili najslabijih elemenata jednog skupa u odnosu na jednu ili više primenjivih ili u odnosu na jedan zajednički sintetički kriterijum. Problemi ove vrste vezani su u svakodnevnoj praksi za mnogobrojne društvene, naučne i privredne aktivnosti.

Rešavanje ovih problema, u vidu identifikacije grupe najboljih ili grupe najslabijih elemenata posmatranog skupa elemenata na osnovu nekog kriterijuma X, biće znatno olakšano ako se taj kriterijum može kvantifikovati i tako uspostaviti redosledna klasifikacija elemenata posmatranog skupa. Tada se problem svodi na to da se u tom redosledu povuku granice koje bi izdvojile grupu najboljih odnosno grupu najslabijih elemenata.

Ovako definisan problem je u opštem slučaju neodređen, jer ako ne postoje neki dopunski uslovi, broj mogućih rešenja će biti jednak broju elemenata umanjenom za jedan.

U radu je dato nekoliko postupaka za smanjivanje stepena neodređenosti tako da dobijeni redosled za dati kriterijum možemo koristiti odgovarajuću matricu sličnosti i preko Sorensen-ovog dendrograma definisati hijerarhijsku klasifikaciju posmatranog skupa elemenata. Ova hijerarhija nam omogućava da putem metode restriktivnih klasa obrazujemo jedan redukovani monotoni niz delova toga skupa čije je zajedničko jezgro njegov prvi (najbolji) odnosno poslednji (najslabiji) elemenat. Na taj način biće smanjen stepen neodređenosti, jer je redukovan broj alternativnih rešenja a međusobne razlike između delova mnogo su jasnije izražene nego između susednih elemenata u redosledu pa je mnogo lakše i doneti odgovarajuće odluke.

Na analogan način se mogu rešiti i problemi kod kojih se redosled određuje preko jednog a dendrogram preko nekog drugog kriterijuma.

U praksi se dešava da se unapred, iz nekih posebnih razloga, odluči da neki elementi treba da uđu u grupu najslabijih. Da bi se i tada došlo do objektivnog i pravednog rešenja, koristiće se uz Sorensen-ov dendrogram i metod restriktivnih klasa i tako dobiti grupa najslabijih koja će sadržavati i one unapred uključene elemente. Rešenja će tada biti jedinstvena.

Takođe, posmatran je slučaj kada je redosled elemenata određen preko jednog kriterijuma i kada se skup alternativnih rešenja podele

na grupu najslabijih i grupu ostalih elemenata, odreduje preko jednog multidimenzionalnog kriterijuma. Ovaj problem je rešen optimalizacijom odgovarajućih koeficijenata separabilnosti.

Najzad, danas se najčešće susrećemo sa problemima u kojima se redosled elemenata vrši preko nekog faktora koji se kvantitativno iskazuje preko jednog sintetičkog indikatora izvedenog preko datog niza pokazatelja. Metod koji uključuje Sorensen-ov dendrogram, omogućiće nam da dođemo do preciznijih rezultata nego metod koeficijenata separabilnosti.

EMPIRICAL RESEARCH INTO GERMAN CODETERMINATION: PROBLEMS AND PERSPECTIVES

*Hans G. NUTZINGER**

I. INTRODUCTION: ORIGINS AND CONCEPTS

I.1 *Historical overview*

The idea of a constitutional limitation of private property rights — and especially of the right to direct other people's work derived from this property — has a long tradition in Germany, starting as early as in the National Assembly of Frankfurt in 1848 *(Paulskirche)*. The development of an institutionalized employee „codetermination" as a modification (or, as property rights theorists would prefer to call it, "attenuation"[1]) of property rights with regard to the use of the means of production has to be seen against the background of the specific economic and political development of Germany, above all in the late 19th and the early 20th century.[2]

The specific features of the German course of events in the frame of the general process of industrialization in Western Europe and Northern America have to be seen mainly in the following characteristics:

— In contrast to the leading European powers in the middle of the 19th century, especially England and France, Germany had not yet overcome the historical splintering of the territory, and its way to a modern nation-state was further complicated by the emerging conflict between Prussia and Austria.

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[1] See section III below for a critical examination of the so-called "attenuation" aspect of codetermination.

[2] For an overview of the historical development, see Nutzinger (1981) with further references.

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