

LIMITATIONS ON THE PROCEDURE OF ROUNDING

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In the literature on integer linear models, limitations on the application of rounding the optimal solution to the relaxed problem are set. This note gives a counterexample resulting from the consideration of a production program for one of those limitations.

In his excellent book on integer programming, Taha [2] specifies two limitations on the procedure of rounding basic variable values in the continuous optimum.

1. If a *feasible* solution is obtained by rounding, one should not be under the illusion that such a solution is optimal or even close to optimal, and

2. Any integer model having an *original equality* constraint can never yield a feasible integer solution through rounding.

Taha illustrated these statements using a numerical example. Here we shall give a counterexample:

$$\max z = x_1 + 3x_2 + 3x_3 + 2x_4 + 4x_5 + 2x_6$$

$$2x_1 + x_2 \leq 6$$

$$2x_3 + 3x_4 \leq 12$$

$$x_5 + 2x_6 \leq 8$$

$$x_1 - x_2 + x_3 - x_4 + x_5 - x_6 = 0$$

$$x_j \geq 0, (j = 1, 2, \dots, 6) \text{ and integer}$$

When the integrality condition is neglected, the problem has the following optimal basic solution:  $x_1 = 0, x_2 = 6, x_3 = 1, 2, x_4 = 3, 2, x_5 = 8$  and  $x_6 = 0$  with  $z = 60$ . The rounded solution ( $x_1 = 0, x_2 = 6, x_3 = 1, x_4 = 3, x_5 = 8, x_6 = 0$ ) is not only feasible, but so is the optimal solution to the problem. The value of this solution is 59.

Our statement can be easily verified by applying the integer from method to the problem. While doing this, it is interesting that both cutting plane coefficient vectors ( $f_{10}, f_{11}, \dots, f_{1, n+m}$ ), which can be generated, are equal to the vector: (0,2, 0,8, 0,0,0,0, 0,2, 0,6, 0,2, 0,4, 0,6, 0).

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As these vectors, according to Gomory [1], p.275, are elements of the additive group, the sum of their module 1 components, that is (0,4, 0,6, 0,0,0,0, 0,4, 0,2, 0,4, 0,8, 0,2, 0), enabled us to form such a cutting plane that we obtained the above integer optimum only in one iteration by the dual simplex method (with  $-f_{18} = 0,4$  as the pivot element).

Our counterexample is concerned, of course, only with the limitation 2. Consequently, one can find such a problem of integer programming where the solution is obtained by the procedure of rounding the optimal solution to the relaxed problem.

Received: 25. 9. 1980

Revised: 10. 10. 1980

#### REFERENCES

1. R.E. Gomory, "An Algorithm for Integer Solutions to Linear Programs", in *Recent Advances in Mathematical Programming*, R.E. Graves and P. Wolfe (ed.), McGraw — Hill, New York, 1963, pp. 269—302.
2. H.A. Taha, *Integer Programming. Theory, Applications, and Computations*, Academic Press, New York, 1975.

#### O OGRANICENJIMA NA PROCEDURU ZAOKRUŽIVANJA

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#### Rezime

U literaturi o cjelobrojnim linearnim modelima postavljena su ograničenja na primjenu zaokruživanja optimalnog rješenja relaksiranog problema. Tako npr. H.A. Taha u svojoj izvrsnoj knjizi o cjelobrojnom programiranju specificira dvije limitacije na proceduru zaokruživanja bazičnih varijabla u kontinuiranom optimumu.

1. Ako je neko moguće rješenje dobiveno zaokruživanjem, ne treba imati iluziju da je takvo rješenje optimalno ili čak blizu optimalnog, i

2. Cjelobrojni model koji ima originalnu jednadžbu kao ograničenje nikad ne dopušta moguće cjelobrojno rješenje putem zaokruživanja.

Na tu drugu limitaciju dat je u ovoj bilješci jedan kontraprimjer koji je proizašao iz razmatranja jednog proizvodnog programa. Pokazano je da se može naći problem cjelobrojnog programiranja za kojega se rješenje dobije procedurom zaokruživanja optimalnog rješenja relaksiranog problema. Druga limitacija, dakle, općenito ne stoji.

#### SELF-MANAGEMENT IN CHINA?<sup>1</sup>

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#### INTRODUCTION

China has been noticeably absent from the debates on self-management theory and its practical application. The reasons for this cannot be its size — or even the size of the collectively-owned sector of the economy which comprises over 250,000 industrial enterprises and about 50,000 communes, formed from smaller production brigades and teams. The fact that it is a 'Third World' country cannot explain its absence, for others such as Peru and Algeria have received significant attention. Neither could the argument that it is a negative example of trends towards self-management for both capitalist and state socialist societies in Europe and elsewhere, quoted as evidence by proponents of self-management<sup>2</sup>. The explanation for China's absence is most probably a result of both a concern with more advanced industrialized economies and the past explicit hostility of the Chinese towards theories of self-management as well as to the countries in which such developments have taken place, notably Yugoslavia. For example, after an initial cautious approach in the mid-1950's, the dominant Chinese views were of outright hostility — until the past few years. Now the situation is very different as the statements below from 1956 to 1980 make clear!

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<sup>1</sup> This article is a slightly revised version of a paper presented at the Second International Conference on the Economics of Workers' Self-Management in July 1980 at Boğaziçi University, Istanbul, Turkey. I would like to thank those who commented on it at the Conference or afterwards (in particular Paul Hare) as well as those who made suggestions based on a previous article in *China Now*. Some — but not all — of these have been incorporated into this article as have any significant changes or developments in policy from July to mid-November 1980. I would also like to thank Joan Wright and Beverley Shields for their assistance in typing various versions of the article.

<sup>2</sup> An examination of the index of Vanek's collection on *Self-Management* (1975) shows that the USSR has 79 references, Yugoslavia 62, Peru 60 and France 20 indicating that advanced capitalist and socialist countries are considered as well as those from the Third World, including examples of economic systems which are not self-managed. China, however, is not mentioned.